PART I

CHAPTER 1
Functions and Their Graphs

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Section 1.1 Lines in the Plane

You should know the following important facts about lines.

- The graph of $y = mx + b$ is a straight line. It is called a linear equation.
- The slope of the line through (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}.$$

- (a) If $m > 0$ the line rises from left to right. (b) If $m = 0$, the line is horizontal.
  (c) If $m < 0$,, the line falls from left to right. (d) If m is undefined, the line is vertical.
- Equations of Lines
  (a) Slope-Intercept: $y = mx + b$
  (b) Point-Slope: $y - y_1 = m(x - x_1)$
  (c) Two-Point: $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$
  (d) General: $Ax + By + c = 0$
  (e) Vertical: $x = a$
  (f) Horizontal: $y = b$
- Given two distinct nonvertical lines

$$L_1: y = m_1x + b_1 \quad \text{and} \quad L_2: y = m_2x + b_2$$

  (a) L_1 is parallel to L_2 if and only if $m_1 = m_2$ and $b_1 \neq b_2$.
  (b) L_1 is perpendicular to L_2 if and only if $m_1 = -1/m_2$.

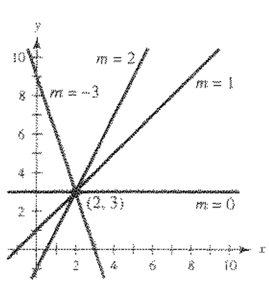
Vocabulary Check

1. (a) iii (b) i (c) v (d) ii (e) iv
2. slope
3. parallel
4. perpendicular
5. linear extrapolation

1. (a) $m = \frac{2}{3}$. Since the slope is positive, the line rises. Matches L_2.
   (b) m is undefined. The line is vertical. Matches L_3.
   (c) $m = -2$. The line falls. Matches L_1.

2. (a) $m = 0$. The line is horizontal. Matches L_2.
   (b) $m = -\frac{3}{4}$. Because the slope is negative, the line falls. Matches L_1.
   (c) $m = 1$. Because the slope is positive, the line rises. Matches L_3.

3.



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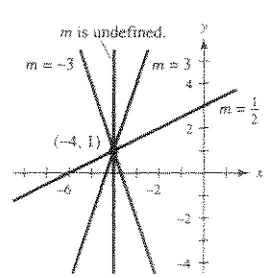
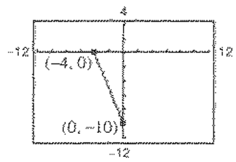


5. Slope $= \dfrac{\text{rise}}{\text{run}} = \dfrac{3}{2}$

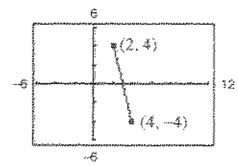
6. The line appears to go through $(0, 8)$ and $(2, 0)$.

Slope $= \dfrac{8 - 0}{0 - 2} = -4$

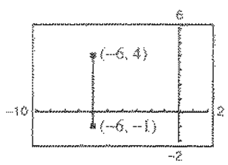
7. Slope $= \dfrac{0 - (-10)}{-4 - 0} = \dfrac{10}{-4} = -\dfrac{5}{2}$



8. Slope $= \dfrac{-4 - 4}{4 - 2} = -4$

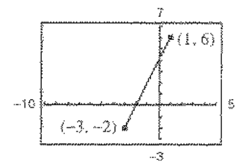


9.



Slope is undefined.

10. Slope $= \dfrac{6 - (-2)}{1 - (-3)} = \dfrac{8}{4} = 2$



11. Since $m = 0$, y does not change. Three points are $(0, 1)$, $(3, 1)$, and $(-1, 1)$.

12. Since $m = 0$, y does not change. Three additional points: $(0, -2)$, $(1, -2)$, $(4, -2)$.

13. Since m is undefined, x does not change and the line is vertical. Three points are $(1, 1)$, $(1, 2)$, and $(1, 3)$.

14. Because m is undefined, x does not change. Three other points are: $(-4, 0)$, $(-4, 3)$, $(-4, 5)$.

15. Since $m = -2$, y decreases 2 for every unit increase in x. Three points are $(1, -11)$, $(2, -13)$, and $(3, -15)$.

16. Since $m = 2$, y increases 2 for every unit increase in x. Three points are: $(-4, 6)$, $(-3, 8)$, and $(-2, 10)$.

17. Since $m = \frac{1}{2}$, y increases 1 for every increase of 2 in x. Three points are $(9, -1)$, $(11, 0)$,and $(13, 1)$.

18. Since $m = -\frac{1}{2}$, y decreases 1 for every increase of 2 units in x. Three points are $(1, -7)$, $(3, -8)$, $(5, -9)$.

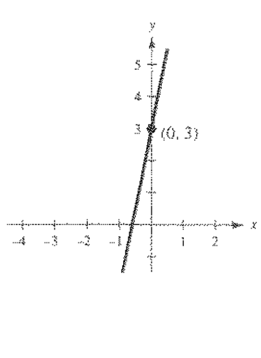
19. $5x - y + 3 = 0$

$$y = 5x + 3$$

(a) Slope: $m = 5$

y-intercept: $(0, 3)$

(b)



20. $2x + 3y - 9 = 0$

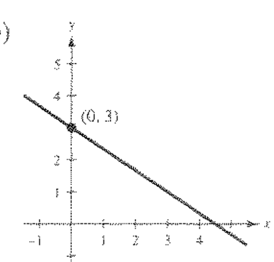
$$3y = -2x + 9$$

$$y = -\tfrac{2}{3}x + 3$$

(a) Slope: $m = -\frac{2}{3}$

y-intercept: $(0, 3)$

(b)



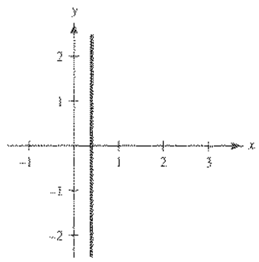
21. $5x - 2 = 0$

$x = \frac{2}{5}$

(a) Slope: undefined

No y-intercept

(b)



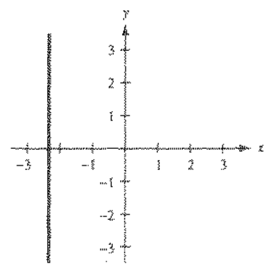
22. $3x + 7 = 0$

$x = -\frac{7}{3}$

(a) Slope: undefined

y-intercept: none

(b)



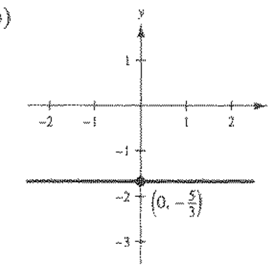
23. $3y + 5 = 0$

$y = -\frac{5}{3}$

(a) Slope: $m = 0$

y-intercept: $\left(0, -\frac{5}{3}\right)$

(b)



24. $-11 - 8y = 0$

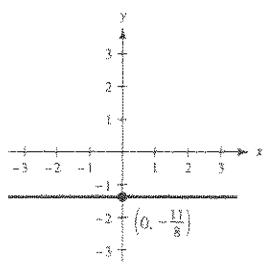
$8y = -11$

$y = -\frac{11}{8}$

(a) Slope: $m = 0$

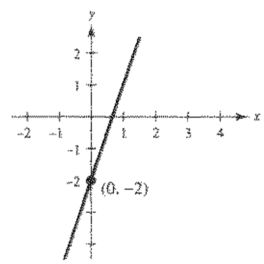
y-intercept: $\left(0, -\frac{11}{8}\right)$

(b)



25. $y + 2 = 3(x - 0)$

$y = 3x - 2 \implies 3x - y - 2 = 0$

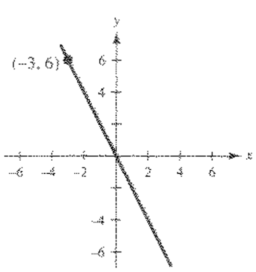


26. (a) $m = -2, \quad (-3, 6)$

$y - 6 = -2(x + 3)$

$y = -2x$

$2x + y = 0$

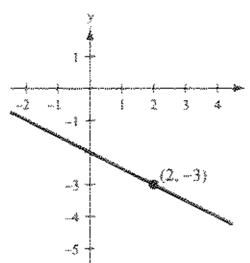


27. $y - (-3) = -\frac{1}{2}(x - 2)$

$y + 3 = -\frac{1}{2}x + 1$

$2y + 4 = -x$

$x + 2y + 4 = 0$

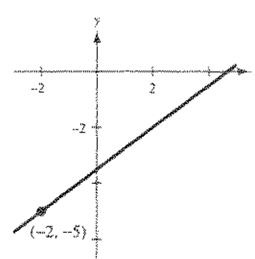


28. $m = \frac{3}{4}, (-2, -5)$

$y + 5 = \frac{3}{4}(x + 2)$

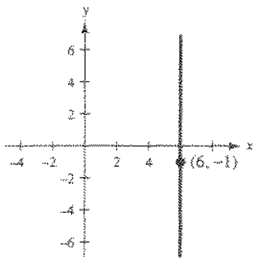
$4y + 20 = 3x + 6$

$0 = 3x - 4y - 14$



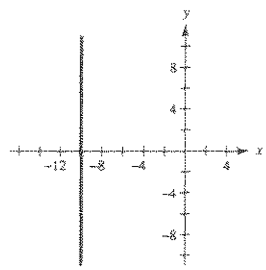
29. $x = 6$

$x - 6 = 0$



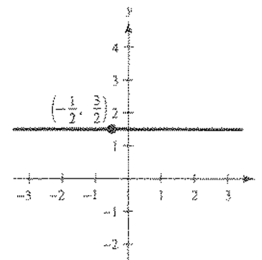
30. m undefined. Line is vertical.

$x + 10 = 0$



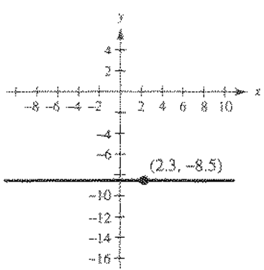
31. $y - \frac{3}{2} = 0\left(x + \frac{1}{2}\right)$

$y - \frac{3}{2} = 0$ horizontal line



32. $m = 0$. Line is horizontal.

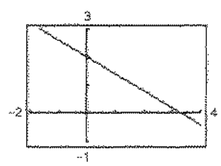
$y + 8.5 = 0$



33. $y + 1 = \dfrac{5 + 1}{-5 - 5}(x - 5)$

$y = -\dfrac{3}{5}(x - 5) - 1$

$y = -\dfrac{3}{5}x + 2$

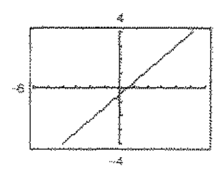


34. $(4, 3), (-4, -4)$

$y - 3 = \dfrac{-4 - 3}{-4 - 4}(x - 4)$

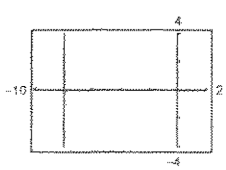
$y - 3 = \dfrac{7}{8}(x - 4)$

$y = \dfrac{7}{8}x - \dfrac{1}{2}$



35. Since both points have $x = -8$, the slope is undefined.

$x = -8$



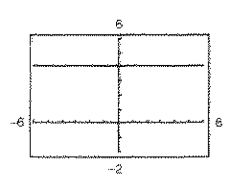
36. $(-1, 4), (6, 4)$

$y - 4 = \dfrac{4 - 4}{6 - (-1)}(x + 1)$

$y - 4 = 0(x + 1)$

$y - 4 = 0$

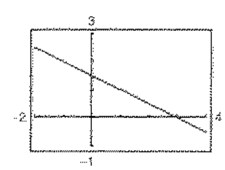
$y = 4$



37. $y - \dfrac{1}{2} = \dfrac{\frac{5}{4} - \frac{1}{2}}{\frac{1}{2} - 2}(x - 2)$

$y = -\dfrac{1}{2}(x - 2) + \dfrac{1}{2}$

$y = -\dfrac{1}{2}x + \dfrac{3}{2}$



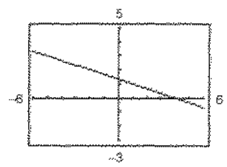
38. $(1, 1), \left(6, -\frac{2}{3}\right)$

$$y - 1 = \frac{-\frac{2}{3} - 1}{6 - 1}(x - 1)$$

$$y - 1 = -\frac{1}{3}(x - 1)$$

$$y - 1 = -\frac{1}{3}x + \frac{1}{3}$$

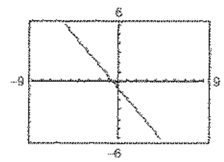
$$y = -\frac{1}{3}x + \frac{4}{3}$$



39. $y + \frac{3}{5} = \frac{-\frac{9}{5} + \frac{3}{5}}{\frac{9}{10} + \frac{1}{10}}\left(x + \frac{1}{10}\right)$

$$y + \frac{3}{5} = -\frac{6}{5}\left(x + \frac{1}{10}\right)$$

$$y = -\frac{6}{5}x - \frac{18}{25}$$



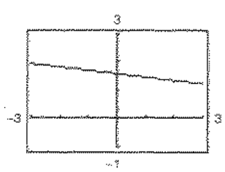
40. $\left(\frac{3}{4}, \frac{3}{2}\right), \left(-\frac{4}{3}, \frac{7}{4}\right)$

$$y - \frac{3}{2} = \frac{\frac{7}{4} - \frac{3}{2}}{-\frac{4}{3} - \frac{3}{4}}\left(x - \frac{3}{4}\right)$$

$$y - \frac{3}{2} = -\frac{3}{25}\left(x - \frac{3}{4}\right)$$

$$y - \frac{3}{2} = -\frac{3}{25}x + \frac{9}{100}$$

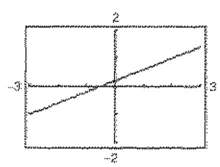
$$y = -\frac{3}{25}x + \frac{159}{100}$$



41. $y - 0.6 = \frac{-0.6 - 0.6}{-2 - 1}(x - 1)$

$$y = 0.4(x - 1) + 0.6$$

$$y = 0.4x + 0.2$$

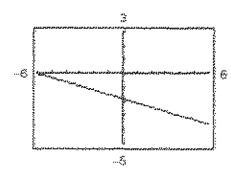


42. $(-8, 0.6), (2, -2.4)$

$$y - 0.6 = \frac{-2.4 - 0.6}{2 - (-8)}(x + 8)$$

$$y - 0.6 = -\frac{3}{10}(x + 8)$$

$$y = -\frac{3}{10}x - \frac{9}{5}$$



43. The slope is $\frac{-3 - (-7)}{1 - (-1)} = \frac{4}{2} = 2.$

$$y - (-3) = 2(x - 1)$$

$$y + 3 = 2x - 2$$

$$y = 2x - 5$$

44. The slope is $\frac{-1 - \frac{3}{2}}{4 - (-1)} = \frac{-\frac{5}{2}}{5} = \frac{-1}{2}.$

$$y - (-1) = -\frac{1}{2}(x - 4)$$

$$y + 1 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 1$$

45. Using the points (2004, 28,500) and (2006, 32,900), you have

$$m = \frac{32{,}900 - 28{,}500}{2006 - 2004} = \frac{4400}{2} = 2200$$

$$S - 28{,}500 = 2200(t - 2004)$$

$$S = 2200t - 4{,}380{,}300.$$

When $t = 2008$,

$$S = 2200(2008) - 4{,}380{,}300 = \$37{,}300.$$

46. Using the points (2004, 25,000) and (2006, 27,500), you have

$$m = \frac{27{,}500 - 25{,}000}{2006 - 2004} = \frac{2500}{2} = 1250$$

$$S - 25{,}000 = 1250(t - 2004)$$

$$S = 1250t - 2{,}480{,}000.$$

When $t = 2008$,

$$S = 1250(2008) - 2{,}480{,}000 = \$30{,}000.$$

47. $x - 2y = 4$

$$-2y = -x + 4$$

$$y = \frac{1}{2}x - 2$$

Slope: $\frac{1}{2}$

y-intercept: $(0, -2)$

The graph passes through $(0, -2)$ and rises 1 unit for each horizontal increase of 2.

48. $3x + 4y = 1$

$$4y = -3x + 1$$

$$y = \frac{-3}{4}x + \frac{1}{4}$$

Slope: $-\frac{3}{4}$

y-intercept: $\left(0, \frac{1}{4}\right)$

The line slopes downward and passes through the point $\left(0, \frac{1}{4}\right)$.

49. $x = -6$

slope is undefined

no y-intercept

The line is vertical and passes through $(-6, 0)$.

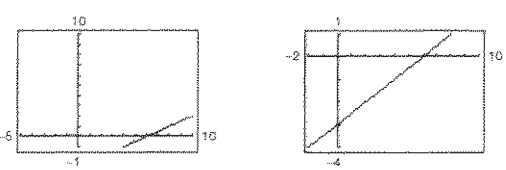
50. $y = 12$

Slope: 0

y-intercept: $(0, 12)$

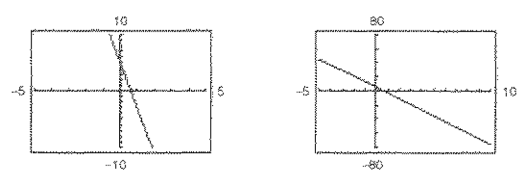
The line is horizontal and passes through $(0, 12)$.

51. $y = 0.5x - 3$



The second setting shows the x- and y-intercepts more clearly.

52.

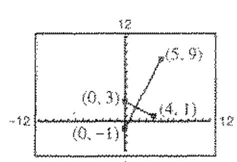


The first setting shows the x- and y-intercepts more clearly.

53. $m_{L_1} = \frac{9 + 1}{5 - 0} = 2$

$$m_{L_2} = \frac{1 - 3}{4 - 0} = -\frac{1}{2} = -\frac{1}{m_{L_1}}$$

L_1 and L_2 are perpendicular.



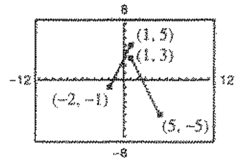
54. L_1: $(-2, -1), (1, 5)$

$$m_1 = \frac{5-(-1)}{1-(-2)} = \frac{6}{3} = 2$$

L_2: $(1, 3), (5, -5)$

$$m_2 = \frac{-5-3}{5-1} = \frac{-8}{4} = -2$$

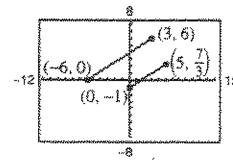
The lines are neither parallel nor perpendicular.



55. $m_{L_1} = \dfrac{0-6}{-6-3} = \dfrac{2}{3}$

$$m_{L_2} = \frac{\frac{7}{3}+1}{5-0} = \frac{2}{3} = m_{L_1}$$

L_1 and L_2 are parallel.



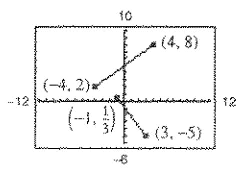
56. L_1: $(4, 8), (-4, 2)$

$$m_1 = \frac{2-8}{-4-4} = \frac{-6}{-8} = \frac{3}{4}$$

L_2: $(3, -5), \left(-1, \dfrac{1}{3}\right)$

$$m_2 = \frac{(1/3)-(-5)}{-1-3} = \frac{16/3}{-4} = -\frac{4}{3}$$

The lines are perpendicular.



57. $4x - 2y = 3$

$$y = 2x - \frac{3}{2}$$

Slope: $m = 2$

(a) $y - 1 = 2(x - 2)$

$$y = 2x - 3$$

(b) $y - 1 = -\dfrac{1}{2}(x - 2)$

$$y = -\frac{1}{2}x + 2$$

58. $x + y = 7$

$$y = -x + 7$$

Slope: $m = -1$

(a) $m = -1, (-3, 2)$

$$y - 2 = -1(x + 3)$$

$$y = -x - 1$$

(b) $m = 1, (-3, 2)$

$$y - 2 = 1(x + 3)$$

$$y = x + 5$$

59. $3x + 4y = 7$

$$y = -\tfrac{3}{4}x + \tfrac{7}{4}$$

Slope: $m = -\frac{3}{4}$

(a) $y - \frac{7}{8} = -\frac{3}{4}\left(x + \frac{2}{3}\right)$

$$y = -\tfrac{3}{4}x + \tfrac{3}{8}$$

(b) $y - \frac{7}{8} = \frac{4}{3}\left(x + \frac{2}{3}\right)$

$$y = \tfrac{4}{3}x + \tfrac{127}{72}$$

60. $6x + 2y = 9$

$$2y = -6x + 9$$

$$y = -3x + \tfrac{9}{2}$$

Slope: $m = -3$

(a) $m = -3, (-3.9, -1.4)$

$$y + 1.4 = -3(x + 3.9)$$

$$y = -3x - 13.1$$

(b) $m = \frac{1}{3}, (-3.9, -1.4)$

$$y + 1.4 = \tfrac{1}{3}(x + 3.9)$$

$$y = \tfrac{1}{3}x - \tfrac{1}{10}$$

61. $x - 4 = 0$ vertical line

slope not defined

(a) $x - 3 = 0$ passes through $(3, -2)$

(b) $y = -2$ passes through $(3, -2)$ and is horizontal

62. $y + 2 = 0$

$$y = -2$$

Slope: $m = 0$

(a) $m = 0, \quad (-4, 1)$

$$y = 1$$

(b) m undefined (vertical line)

$$x = -4$$

63. The slope is 2 and $(-1, -1)$ lies on the line. Hence,

$$y - (-1) = 2(x - (-1))$$
$$y + 1 = 2(x + 1)$$
$$y = 2x + 1.$$

64. The slope is -2 and $(-1, 1)$ lies on the line. Hence,

$$y - 1 = -2(x - (-1))$$
$$y - 1 = -2(x + 1)$$
$$y = -2x - 1.$$

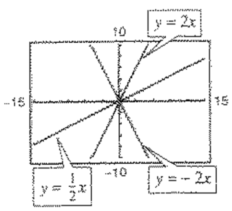
65. The slope of the given line is 2. Then l has slope $-\frac{1}{2}$. Hence,

$$y - 2 = -\tfrac{1}{2}(x - (-2))$$
$$y - 2 = -\tfrac{1}{2}(x + 2)$$
$$y = -\tfrac{1}{2}x + 1.$$

66. The slope of the given line is 3. Then l has slope $-\frac{1}{3}$. Hence,

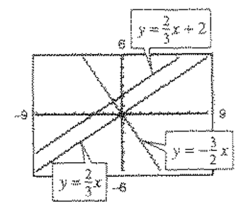
$$y - 5 = -\tfrac{1}{3}(x - (-3))$$
$$y - 5 = -\tfrac{1}{3}(x + 3)$$
$$y = -\tfrac{1}{3}x + 4.$$

67. (a) $y = 2x$ (b) $y = -2x$ (c) $y = \frac{1}{2}x$



(b) and (c) are perpendicular.

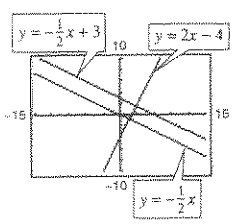
68. L_1: $y = \frac{2}{3}x$; L_2: $y = -\frac{3}{2}x$; L_3: $y = \frac{2}{3}x + 2$



L_1 is parallel to L_3. L_2 is perpendicular to L_1 and L_3.

69. (a) $y = -\frac{1}{2}x$ (b) $y = -\frac{1}{2}x + 3$

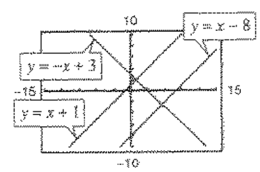
(c) $y = 2x - 4$



(a) and (b) are parallel.

(c) is perpendicular to (a) and (b).

70. L_1: $y = x - 8$; L_2: $y = x + 1$; L_3: $y = -x + 3$



L_1 is parallel to L_2. L_3 is perpendicular to L_1 and L_2.

71. (a)

Years	*Slope*
1995–1996	$0.69 - 0.91 = -0.22$
1996–1997	$0.57 - 0.69 = -0.12$
1997–1998	$0.74 - 0.57 = 0.17$
1998–1999	$1.60 - 0.74 = 0.86$
1999–2000	$0.82 - 1.60 = -0.78$
2000–2001	$0.92 - 0.82 = 0.10$
2001–2002	$0.20 - 0.92 = -0.72$
2002–2003	$0.00 - 0.20 = -0.20$
2003–2004	$0.31 - 0.00 = 0.31$

Greatest increase: 1998–1999 (0.86)

Greatest decrease: 1999–2000 (-0.78)

(b) $(5, 0.91)$, $(14, 0.31)$:

$$y - 0.91 = \frac{0.31 - 0.91}{14 - 5}(x - 5)$$

$$y = -\frac{1}{15}(x - 5) + \frac{91}{100} = -\frac{1}{15}x + \frac{373}{300}$$

$$y \approx -0.07x + 1.24$$

(c) Between 1995 and 2004, the earnings per share decreased at the rate of 0.07 per year.

(d) For 2010, $x = 20$ and $y = -0.07(20) + 1.24 = -0.16$, which is reasonable.

72. (a)

Years	*Slope*
1995–1996	$13.1 - 13.2 = -0.1$
1996–1997	$13.2 - 13.1 = 0.1$
1997–1998	$12.6 - 13.2 = -0.6$
1998–1999	$12.9 - 12.6 = 0.3$
1999–2000	$14.4 - 12.9 = 1.5$
2000–2001	$14.1 - 14.4 = -0.3$
2001–2002	$13.9 - 14.1 = -0.2$
2002–2003	$15.1 - 13.9 = 1.2$
2003–2004	$18.4 - 15.1 = 3.3$

Greatest increase: 2003–2004 (3.3)

Smallest increase: 1996–1997 (0.1)

(b) (5, 13.2), (14, 18.4)

$$y - 13.2 = \frac{18.4 - 13.2}{14 - 5}(x - 5)$$

$$y = \frac{26}{45}x + \frac{464}{45}$$

$$y \approx 0.58x + 10.31$$

(c) Between 1995 and 2004, the sales (in billions of dollars) increased at the rate of 0.58 per year.

(d) For 2010, $x = 20$ and $y = 0.58(20) + 10.31 = 21.91$ (billion), which seems reasonable.

73. $\dfrac{\text{rise}}{\text{run}} = \dfrac{3}{4} = \dfrac{x}{\frac{1}{2}(32)}$

$$\frac{3}{4} = \frac{x}{16}$$

$$4x = 48$$

$$x = 12$$

The maximum height in the attic is 12 feet.

74. Slope $= \dfrac{\text{rise}}{\text{run}}$

$$\frac{-12}{100} = \frac{-2000}{x}$$

$$-12x = (-2000)(100)$$

$$x = 16{,}666\tfrac{2}{3} \text{ ft} \approx 3.16 \text{ miles}$$

75. $(6, 2540)$, $m = 125$

$$V - 2540 = 125(t - 6)$$

$$V = 125t + 1790$$

76. $(6, 156)$, $m = 4.50$

$$V - 156 = 4.50(t - 6)$$

$$V = 4.50t + 129$$

77. $(6, 20{,}400)$, $m = -2000$

$$V - 20{,}400 = -2000(t - 6)$$

$$V = -2000t + 32{,}400$$

78. $(6, 245{,}000)$, $m = -5600$

$$V - 245{,}000 = -5600(t - 6)$$

$$V = -5600t + 278{,}600$$

79. The slope is $m = -10$. This represents the decrease in the amount of the loan each week. Matches graph (b).

80. The y-intercept is 12.5 and the slope is 1.5, which represents the increase in hourly wage per unit produced. Matches graph (c).

81. The slope is $m = 0.35$. This represents the increase in travel cost for each mile driven. Matches graph (a).

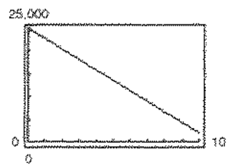
82. The y-intercept is 600 and the slope is -100, which represents the decrease in the value of the word processor each year. Matches graph (d).

83. (a) (0, 25,000), (10, 2000)

$$V - 25{,}000 = \frac{2000 - 25{,}000}{10 - 0}(t - 0)$$

$$V - 25{,}000 = -2300t$$

$$V = -2300t + 25{,}000$$

(b) 

t	0	1	2	3	4	5	6	7	8	9	10
V	25,000	22,700	20,400	18,100	15,800	13,500	11,200	8900	6600	4300	2000

(c) $t = 0$: $V = -2300(0) + 25{,}000 = 25{,}000$

$t = 1$: $V = -2300(1) + 25{,}000 = 22{,}700$

etc.

84. (a) Using the points (0, 32) and (100, 212), we have

$$m = \frac{212 - 32}{100 - 0} = \frac{180}{100} = \frac{9}{5}$$

$$F - 32 = \frac{9}{5}(C - 0)$$

$$F = \frac{9}{5}C + 32.$$

(b) $F = \frac{9}{5}C + 32$

$F = 0°$: $0 = \frac{9}{5}C + 32$

$-32 = \frac{9}{5}C$

$-17.8 \approx C$

$C = 10°$: $F = \frac{9}{5}(10) + 32$

$F = 18 + 32$

$F = 50$

$F = 90°$: $90 = \frac{9}{5}C + 32$

$58 = \frac{9}{5}C$

$32.2 \approx C$

$C = -10°$: $F = \frac{9}{5}(-10) + 32$

$F = -18 + 32$

$F = 14$

$F = 68°$: $68 = \frac{9}{5}C + 32$

$36 = \frac{9}{5}C$

$20 = C$

$C = 177°$: $F = \frac{9}{5}(177) + 32$

$F = 318.6 + 32$

$F = 350.6$

C	−17.8°	−10°	10°	20°	32.2°	177°
F	0°	14°	50°	68°	90°	350.6°

85. (a) $C = 36{,}500 + 5.25t + 11.50t$

$= 16.75t + 36{,}500$

(b) $R = 27t$

(c) $P = R - C$

$= 27t - (16.75t + 36{,}500)$

$= 10.25t - 36{,}500$

(d) $0 = 10.25t - 36{,}500$

$36{,}500 = 10.25t$

$t \approx 3561$ hours

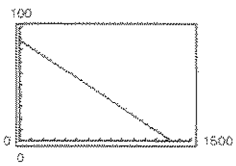
86. (a) (580, 50), (625, 47)

$$x - 50 = \frac{47 - 50}{625 - 580}(p - 580)$$

$$x - 50 = \frac{-1}{15}(p - 580)$$

$$x = \frac{-1}{15}p + \frac{266}{3}$$

(b)



If $p = 655$, $x = 45$, units.

Algebraically, $x = -\frac{1}{15}(655) + \frac{266}{3} = 45.$

(c) If $p = 595$, $x = 49$ units.

Algebraically, $x = -\frac{1}{15}(595) + \frac{266}{3} = 49.$

87. (a) $\frac{80{,}124 - 75{,}349}{2005 - 1991} = \frac{4775}{14} \approx 341$ students per year

(b) 1984: $75{,}349 - 341(7) \approx 72{,}962$ students

1997: $75{,}349 + 341(6) \approx 77{,}395$ students

2000: $75{,}349 + 341(9) \approx 78{,}418$ students

(Answers could vary.)

(c) Let $t = 0$ represent 1990.

(1, 75,349), (15, 80,124)

$$y - 75{,}349 = \frac{80{,}124 - 75{,}349}{15 - 1}(t - 1)$$

$$y = \frac{4775}{14}(t - 1) + 75{,}349$$

$$y \approx 341t + 75{,}008$$

The slope 341 represents the annual increase in students. It is positive, indicating that Penn State University increased its students from 1991 to 2005.

88. Answers will vary. The slope is 341 which is equivalent to the rate of change.

89. False. The slopes are different:

$$\frac{4 - 2}{-1 + 8} = \frac{2}{7}$$

$$\frac{7 + 4}{-7 - 0} = -\frac{11}{7}$$

90. False.

The equation of the line joining $(10, -3)$ and $(2, -9)$ is

$$y + 3 = \frac{-9 + 3}{2 - 10}(x - 10)$$

$$y + 3 = \frac{3}{4}(x - 10)$$

$$y = \frac{3}{4}x - \frac{21}{2}.$$

For $x = -12$, $y = \frac{3}{4}(-12) - \frac{21}{2}$

$$= -19.5$$

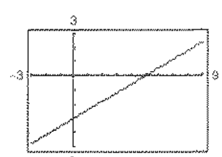
$$\neq \frac{-37}{2}$$

$$= -18.5$$

91. $\frac{x}{5} + \frac{y}{-3} = 1$

$$-3x + 5y + 15 = 0$$

$a = 5$ and $b = -3$ are the x- and y-intercepts.



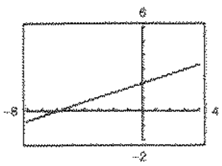
92. $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{-6} + \frac{y}{2} = 1$$

$$y = 2\left(1 + \frac{x}{6}\right)$$

$$y = \frac{x}{3} + 2$$

a and b are the x- and y-intercepts.

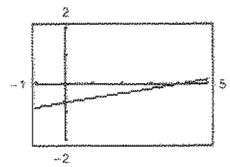


93. $\frac{x}{4} + \frac{y}{-\frac{2}{3}} = 1$

$$-\frac{2}{3}x + 4y = \frac{-8}{3}$$

$$-2x + 12y = -8$$

intercepts: $(4, 0), \left(0, -\frac{2}{3}\right)$

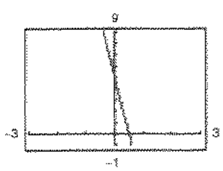


94. $\frac{x}{\frac{1}{2}} + \frac{y}{5} = 1$

$$5x + \frac{1}{2}y = \frac{5}{2}$$

$$10x + y = 5$$

Intercepts: $\left(\frac{1}{2}, 0\right), (0, 5)$



95. $\frac{x}{2} + \frac{y}{3} = 1$

$$3x + 2y - 6 = 0$$

96. $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{-5} + \frac{y}{-4} = 1$$

$$4x + 5y + 20 = 0$$

97. $\frac{x}{-1/6} + \frac{y}{-2/3} = 1$

$$-6x - \frac{3}{2}y = 1$$

$$12x + 3y + 2 = 0$$

98. $\frac{x}{a} + \frac{y}{b} = 1$

$$\frac{x}{3/4} + \frac{y}{4/5} = 1$$

$$\frac{4}{5}x + \frac{3}{4}y = \frac{3}{5}$$

$$16x + 15y - 12 = 0$$

99. The slope is positive and the y-intercept is positive. Matches (a).

100. The slope is negative and the y-intercept is negative. Matches (b).

101. Both lines have positive slope, but their y-intercepts differ in sign. Matches (c).

102. The lines intersect in the first quadrant at a point (x, y) where $x < y$. Matches (a).

103. No. The line $y = 2$ does not have an x-intercept.

104. No. $x = 1$ cannot be written in slope-intercept form because the slope is undefined.

105. Yes. Answers will vary.

106. Yes. Answers will vary.

107. Yes. $x + 20$

108. Yes. $3x - 10x^2 + 1 = -10x^2 + 3x + 1$

109. No. The term $x^{-1} = \dfrac{1}{x}$ causes the expression to not be a polynomial.

110. Yes. $2x^2 - 2x^4 - x^3 + 2 = -2x^4 - x^3 + 2x^2 + 2$

111. No. This expression is not defined for $x = \pm 3$.

112. No.

113. $x^2 - 6x - 27 = (x - 9)(x + 3)$

114. $x^2 - 11x + 28 = (x - 4)(x - 7)$

115. $2x^2 + 11x - 40 = (2x - 5)(x + 8)$

116. $3x^2 - 16x + 5 = (3x - 1)(x - 5)$

117. Answers will vary.

Section 1.2 Functions

- Given a set or an equation, you should be able to determine if it represents a function.
- Given a function, you should be able to do the following.
  (a) Find the domain.
  (b) Evaluate it at specific values.

Vocabulary Check

1. domain, range, function
2. independent, dependent
3. piecewise-defined
4. implied domain
5. difference quotient

1. Yes, it does represent a function. Each domain value is matched with only one range value.

2. No, it is not a function. The domain value of -1 is matched with two output values.

3. No, it does not represent a function. The domain values are each matched with three range values.

4. Yes, it does represent a function. Every domain value is matched with only one range value.

5. Yes, the relation represents y as a function of x. Each domain value is matched with only one range value.

6. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

7. No, it does not represent a function. The input values of 10 and 7 are each matched with two output values.

8. Yes, the table does represent a function. Each input value is matched with only one output value.

9. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
(b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.
(c) Each element of A is matched with exactly one element of B, so it does represent a function.
(d) The element 2 of A is not matched to any element of B, so it does not represent a function.

10. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.

(b) Each element of A is matched with exactly one element of B, so it does represent a function.

(c) This is not a function from A to B (it represents a function from B to A instead).

(d) Each element in A is matched with exactly one element of B, so it does represent a function.

11. Each are functions. For each year there corresponds one and only one circulation.

12. $f(2003) = 7.7$ million newspapers

13. $x^2 + y^2 = 4 \implies y = \pm\sqrt{4 - x^2}$

Thus, y *is not* a function of x. For instance, the values $y = 2$ and -2 both correspond to $x = 0$.

14. $x = y^2 + 1$

$y = \pm\sqrt{x - 1}$

This *is not* a function of x. For example, the values $y = 2$ and $y = -2$ both correspond to $x = 5$.

15. $y = \sqrt{x^2 - 1}$

This *is* a function of x.

16. $y = \sqrt{x + 5}$

This *is* a function of x.

17. $2x + 3y = 4 \implies y = \frac{1}{3}(4 - 2x)$

Thus, y *is* a function of x.

18. $x = -y + 5 \implies y = -x + 5.$

This *is* a function of x.

19. $y^2 = x^2 - 1 \implies y = \pm\sqrt{x^2 - 1}$

Thus, y *is not* a function of x. For instance, the values $y = \sqrt{3}$ and $-\sqrt{3}$ both correspond to $x = 2$.

20. $x + y^2 = 3 \implies y = \pm\sqrt{3 - x}$

Thus, y *is not* a function of x.

21. $y = |4 - x|$

This *is* a function of x.

22. $|y| = 4 - x \implies y = 4 - x$ or $y = -(4 - x)$

Thus, y *is not* a function of x.

23. $x = -7$ does not represent y as a function of x. All values of y correspond to $x = -7$.

24. $y = 8$ is a function of x, a constant function.

25. $f(x) = \dfrac{1}{x + 1}$

(a) $f(4) = \dfrac{1}{(4) + 1} = \dfrac{1}{5}$

(b) $f(0) = \dfrac{1}{(0) + 1} = 1$

(c) $f(4t) = \dfrac{1}{(4t) + 1} = \dfrac{1}{4t + 1}$

(d) $f(x + c) = \dfrac{1}{(x + c) + 1} = \dfrac{1}{x + c + 1}$

26. $g(x) = x^2 - 2x$

(a) $g(2) = (2)^2 - 2(2) = 0$

(b) $g(-3) = (-3)^2 - 2(-3) = 15$

(c) $g(t + 1) = (t + 1)^2 - 2(t + 1) = t^2 - 1$

(d) $g(x + c) = (x + c)^2 - 2(x + c)$

$= x^2 + 2cx + c^2 - 2x - 2c$

27. $f(t) = 3t + 1$

(a) $f(2) = 3(2) + 1 = 7$

(b) $f(-4) = 3(-4) + 1 = -11$

(c) $f(t + 2) = 3(t + 2) + 1 = 3t + 7$

28. $g(y) = 7 - 3y$

(a) $g(0) = 7 - 3(0) = 7$

(b) $g\left(\frac{7}{3}\right) = 7 - 3\left(\frac{7}{3}\right) = 0$

(c) $g(s + 2) = 7 - 3(s + 2)$

$= 7 - 3s - 6 = 1 - 3s$

29. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x + 2) = (x + 2)^2 - 2(x + 2) = x^2 + 2x$

30. $V(r) = \dfrac{4}{3}\pi r^3$

(a) $V(3) = \dfrac{4}{3}\pi(3)^3 = 36\pi$

(b) $V\left(\dfrac{3}{2}\right) = \dfrac{4}{3}\pi\left(\dfrac{3}{2}\right)^3 = \dfrac{4}{3} \cdot \dfrac{27}{8}\pi = \dfrac{9\pi}{2}$

(c) $V(2r) = \dfrac{4}{3}\pi(2r)^3 = \dfrac{32\pi r^3}{3}$

31. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

32. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-8) = \sqrt{(-8) + 8} + 2 = 2$

(b) $f(1) = \sqrt{(1) + 8} + 2 = 5$

(c) $f(x - 8) = \sqrt{(x - 8) + 8} + 2 = \sqrt{x} + 2$

33. $q(x) = \dfrac{1}{x^2 - 9}$

(a) $q(0) = \dfrac{1}{0^2 - 9} = -\dfrac{1}{9}$

(b) $q(3) = \dfrac{1}{3^2 - 9}$ is undefined.

(c) $q(y + 3) = \dfrac{1}{(y + 3)^2 - 9} = \dfrac{1}{y^2 + 6y}$

34. $q(t) = \dfrac{2t^2 + 3}{t^2}$

(a) $q(2) = \dfrac{2(2)^2 + 3}{(2)^2} = \dfrac{8 + 3}{4} = \dfrac{11}{4}$

(b) $q(0) = \dfrac{2(0)^2 + 3}{(0)^2}$ Division by zero is undefined.

(c) $q(-x) = \dfrac{2(-x)^2 + 3}{(-x)^2} = \dfrac{2x^2 + 3}{x^2}$

35. $f(x) = \dfrac{|x|}{x}$

(a) $f(3) = \dfrac{|3|}{3} = 1$

(b) $f(-3) = \dfrac{|-3|}{-3} = -1$

(c) $f(t) = \dfrac{|t|}{t} = \begin{cases} 1 & \text{if } t > 0 \\ -1 & \text{if } t < 0 \end{cases}$

$f(0)$ is undefined.

36. $f(x) = |x| + 4$

(a) $f(4) = |4| + 4 = 8$

(b) $f(-4) = |-4| + 4 = 4 + 4 = 8$

(c) $f(t) = |t| + 4$

37. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

38. $f(x) = \begin{cases} 2x + 5, & x \le 0 \\ 2 - x^2, & x > 0 \end{cases}$

(a) $f(-2) = 2(-2) + 5 = 1$

(b) $f(0) = 2(0) + 5 = 5$

(c) $f(1) = 2 - 1^2 = 1$

39. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

40. $f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 1 - 2x^2, & x > 0 \end{cases}$

(a) $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

(b) $f(0) = 0^2 - 4 = -4$

(c) $f(1) = 1 - 2(1^2) = 1 - 2 = -1$

41. $f(x) = \begin{cases} x + 2, & x < 0 \\ 4, & 0 \le x < 2 \\ x^2 + 1, & x \ge 2 \end{cases}$

(a) $f(-2) = (-2) + 2 = 0$

(b) $f(1) = 4$

(c) $f(4) = 4^2 + 1 = 17$

42. $f(x) = \begin{cases} 5 - 2x, & x < 0 \\ 5, & 0 \le x < 1 \\ 4x + 1, & x \ge 1 \end{cases}$

(a) $f(-2) = 5 - 2(-2) = 9$

(b) $f(\frac{1}{2}) = 5$

(c) $f(1) = 4(1) + 1 = 5$

43. $h(t) = \frac{1}{2}|t + 3|$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

44. $f(s) = \dfrac{|s - 2|}{s - 2}$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	-1	-1	-1	1	1

$$f(0) = \frac{|0 - 2|}{0 - 2} = \frac{2}{-2} = -1$$

$$f(1) = \frac{|1 - 2|}{1 - 2} = \frac{1}{-1} = -1$$

$$f\left(\frac{3}{2}\right) = \frac{\left|\frac{3}{2} - 2\right|}{\frac{3}{2} - 2} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$$

$$f\left(\frac{5}{2}\right) = \frac{\left|\frac{5}{2} - 2\right|}{\frac{5}{2} - 2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$$

$$f(4) = \frac{|4 - 2|}{4 - 2} = \frac{2}{2} = 1$$

45. $f(x) = \begin{cases} -\frac{1}{2}x + 4, & x \le 0 \\ (x - 2)^2, & x > 0 \end{cases}$

x	-2	-1	0	1	2
$f(x)$	5	$\frac{9}{2}$	4	1	0

46. $h(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \ge 3 \end{cases}$

$h(1) = 9 - (1)^2 = 8$

$h(2) = 9 - (2)^2 = 5$

$h(3) = (3) - 3 = 0$

$h(4) = (4) - 3 = 1$

$h(5) = (5) - 3 = 2$

x	1	2	3	4	5
$h(x)$	8	5	0	1	2

47. $f(x) = 15 - 3x = 0$

$$3x = 15$$

$$x = 5$$

48. $f(x) = 5x + 1 = 0$

$$5x = -1$$

$$x = -\frac{1}{5}$$

49. $f(x) = \dfrac{3x - 4}{5} = 0$

$$3x - 4 = 0$$

$$3x = 4$$

$$x = \frac{4}{3}$$

50. $f(x) = \dfrac{2x - 3}{7} = 0$

$$2x - 3 = 0$$

$$2x = 3$$

$$x = \frac{3}{2}$$

51. $f(x) = g(x)$

$$x^2 = x + 2$$

$$x^2 - x - 2 = 0$$

$$(x + 1)(x - 2) = 0$$

$$x = -1 \text{ or } x = 2$$

52. $f(x) = g(x)$

$$x^2 + 2x + 1 = 7x - 5$$

$$x^2 - 5x + 6 = 0$$

$$(x - 3)(x - 2) = 0$$

$$x = 3 \text{ or } x = 2$$

53. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers x.

54. $g(x) = 1 - 2x^2$

Because $g(x)$ is a polynomial, the domain is all real numbers x.

55. $h(t) = \dfrac{4}{t}$

Domain: All real numbers except $t = 0$

56. $s(y) = \dfrac{3y}{y + 5}$

$$y + 5 \neq 0$$

$$y \neq -5$$

The domain is all real numbers $y \neq -5$.

57. $f(x) = \sqrt[3]{x - 4}$

Domain: all real numbers

58. $f(x) = \sqrt[4]{x^2 + 3x}$. $x^2 + 3x = x(x + 3) \geq 0$

Domain: $x \leq -3$ or $x \geq 0$

59. $g(x) = \dfrac{1}{x} - \dfrac{3}{x + 2}$

Domain: All real numbers except

$x = 0, x = -2$

60. $h(x) = \dfrac{10}{x^2 - 2x}$

$$x^2 - 2x \neq 0$$

$$x(x - 2) \neq 0$$

The domain is all real numbers $x \neq 0$ and $x \neq 2$.

61. $g(y) = \dfrac{y + 2}{\sqrt{y - 10}}$

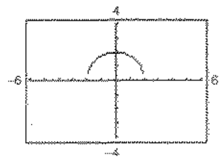
$$y - 10 > 0$$

$$y > 10$$

Domain: all $y > 10$.

62. $f(x) = \dfrac{\sqrt{x + 6}}{6 + x}$. $x + 6 \geq 0$ for numerator, and $x \neq -6$ for denominator. Domain: $x > -6$.

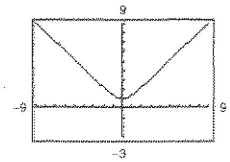
63. $f(x) = \sqrt{4 - x^2}$



Domain: $[-2, 2]$

Range: $[0, 2]$

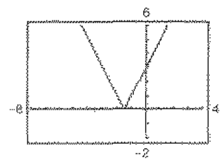
64. $f(x) = \sqrt{x^2 + 1}$



Domain: all real numbers

Range: $1 \le y$

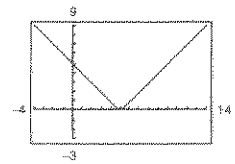
65. $g(x) = |2x + 3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

66. $g(x) = |x - 5|$



Domain: all real numbers

Range: $y \ge 0$

67. $f(x) = x^2$

$\{(-2, 4), (-1, 1), (0, 0), (1, 1), (2, 4)\}$

68. $f(x) = x^2 - 3$

$\{(-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1)\}$

69. $f(x) = |x| + 2$

$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

70. $f(x) = |x + 1|$

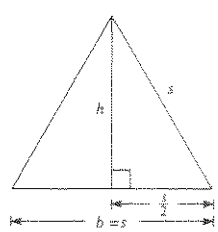
$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

71. $A = \pi r^2, \quad C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

72. $A = \frac{1}{2}bh$, in an equilateral triangle $b = s$ and:



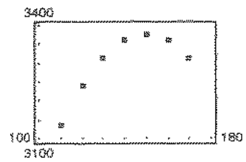
$$s^2 = h^2 + \left(\frac{s}{2}\right)^2$$

$$h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$$

$$h = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \frac{\sqrt{3}s}{2}$$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}s}{2} = \frac{\sqrt{3}s^2}{4}$$

73. (a) According to the table, the maximum profit is 3375 for $x = 150$.

(b) 

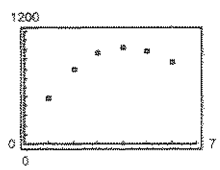
Yes, P is a function of x.

(c) Profit = Revenue − Cost

$$\begin{aligned} &= (\text{price per unit})(\text{number of units}) - (\text{cost})(\text{number of units}) \\ &= [90 - (x - 100)(0.15)]x - 60x \\ &= (105 - 0.15x)x - 60x \\ &= 45x - 0.15x^2, \quad x > 100 \end{aligned}$$

$$P = \begin{cases} 30x, & x \le 100 \\ 45x - 0.15x^2, & x > 100 \end{cases}$$

74. (a) From the table, the maximum volume seems to be 1024, corresponding to $x = 4$.

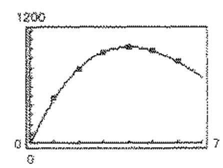
(b) 

Yes, V is a function of x.

(c) $V = \text{length} \times \text{width} \times \text{height}$

$$\begin{aligned} &= (24 - 2x)(24 - 2x)x \\ &= x(24 - 2x)^2 = 4x(12 - x)^2 \end{aligned}$$

Domain: $0 < x < 12$

(d) 

The function is a good fit. Answers will vary.

75. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy.$

Since $(0, y)$, $(2, 1)$ and $(x, 0)$ all lie on the same line, the slopes between any pair of points are equal.

$$\frac{1 - y}{2 - 0} = \frac{1 - 0}{2 - x}$$

$$1 - y = \frac{2}{2 - x}$$

$$y = 1 - \frac{2}{2 - x} = \frac{x}{x - 2}$$

Therefore, $A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{x}{x - 2}\right) = \frac{x^2}{2x - 4}.$

The domain is $x > 2$, since $A > 0$.

76. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

77. (a) $V = (\text{length})(\text{width})(\text{height}) = yx^2$

But, $y + 4x = 108$, or $y = 108 - 4x$.

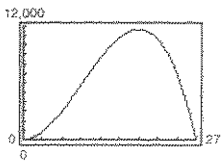
Thus, $V = (108 - 4x)x^2$.

Since $y = 108 - 4x > 0$

$$4x < 108$$

$$x < 27.$$

Domain: $0 < x < 27$

(b) 

(c) The highest point on the graph occurs at $x = 18$. The dimensions that maximize the volume are $18 \times 18 \times 36$ inches.

78. (a) Cost = variable costs + fixed costs

$$C = 12.30x + 98{,}000$$

(b) Revenue = price per unit × number of units

$$R = 17.98x$$

(c) Profit = Revenue − Cost

$$P = 17.98x - (12.30x + 98{,}000)$$

$$P = 5.68x - 98{,}000$$

79. The domain of $-1.97x + 26.3$ is $7 \le x \le 12$.

The domain of $0.505x^2 - 1.47x + 6.3$ is $1 \le x \le 6$.

You can tell by comparing the models to the given data. The models fit the data well on the domains above.

80. $f(5) = 0.505(5^2) - 1.47(5) + 6.3 = 11.575$, which means \$11, 575 in monthly revenue.

81. $f(11) = -1.97(11) + 26.3 = 4.63$

\$4,630 in monthly revenue for November.

82. The values obtained from the model are a close fit for the actual data.

83. $n(t) = \begin{cases} -6.13t^2 + 75.8t + 577, & 0 \le t \le 6 \\ 24.9t + 672, & 6 < t \le 13 \end{cases}$

$t = 0$ corresponds to 1990.

t	0	1	2	3	4	5	6	7	8	9	10	11	12	13
Model	577	647	704	749	782	803	811	846	871	896	921	946	971	996

84. (a) $R = (\text{rate})(\text{number of people})$

$$= [8 - 0.05(n - 80)]n$$

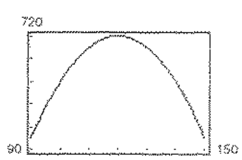
$$= (12 - 0.05n)n = \frac{240n - n^2}{20}, \quad n \ge 80$$

(b)

n	90	100	110	120	130	140	150
$R(n)$	675	700	715	720	715	700	675

The revenue increases, and then decreases.

The maximum revenue occurs when $n = 120$.

(c) 

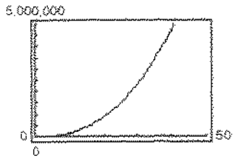
The maximum occurs at $n = 120$.

85. (a) $F(y) = 149.76\sqrt{10}y^{5/2}$

y	5	10	20	30	40
$F(y)$	2.65×10^4	1.50×10^5	8.47×10^5	2.33×10^6	4.79×10^6

(Answers will vary.)

F increases very rapidly as y increases.

(b) 

(c) From the table, $y \approx 22$ ft (slightly above 20). You could obtain a better approximation by completing the table for values of y between 20 and 30.

(d) By graphing $F(y)$ together with the horizontal line $y_2 = 1{,}000{,}000$, you obtain $y \approx 21.37$ feet.

86. (a) $f(2000) \approx 145.6$ billion dollars

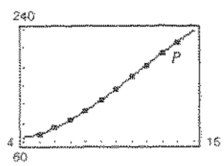
(b) $$\frac{f(2004) - f(1995)}{2004 - 1995} \approx \frac{221 - 72.2}{9} \approx 16.5 \text{ billion dollars/year}$$

This is the average yearly change from 1995 and 2004.

(c)

t	5	6	7	8	9	10	11	12	13	14
$P(t)$	72.4	81.6	94.1	109.1	126.1	144.6	163.9	183.5	202.7	221.0

The model approximates the data well.

(d) 

87.
$$f(x) = 2x$$
$$\frac{f(x+c) - f(x)}{c} = \frac{2(x+c) - 2x}{c}$$
$$= \frac{2c}{c} = 2, \quad c \neq 0$$

88.
$$g(x) = 3x - 1$$
$$g(x+h) = 3(x+h) - 1 = 3x + 3h - 1$$
$$g(x+h) - g(x) = (3x + 3h - 1) - (3x - 1) = 3h$$
$$\frac{g(x+h) - g(x)}{h} = \frac{3h}{h} = 3, h \neq 0$$

89. $f(x) = x^2 - x + 1, \quad f(2) = 3$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - (2+h) + 1 - 3}{h}$$
$$= \frac{4 + 4h + h^2 - 2 - h + 1 - 3}{h}$$
$$= \frac{h^2 + 3h}{h} = h + 3, \quad h \neq 0$$

90. $f(x) = x^3 + x$

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$

$$\begin{aligned} f(x+h) - f(x) &= (x^3 + 3x^2h + 3xh^2 + h^3 + x + h) - (x^3 + x) \\ &= 3x^2h + 3xh^2 + h^3 + h \\ &= h(3x^2 + 3xh + h^2 + 1) \end{aligned}$$

$$\frac{f(x+h) - f(x)}{h} = \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = 3x^2 + 3xh + h^2 + 1, \ h \neq 0$$

91. $f(t) = \frac{1}{t}, \quad f(1) = 1$

$$\frac{f(t) - f(1)}{t - 1} = \frac{\frac{1}{t} - 1}{t - 1} = \frac{1 - t}{t(t-1)} = \frac{-1}{t}, \quad t \neq 1$$

92. $f(x) = \frac{4}{x+1}$

$$f(7) = \frac{4}{7+1} = \frac{1}{2}$$

$$\frac{f(x) - f(7)}{x - 7} = \frac{\frac{4}{x+1} - \frac{1}{2}}{x - 7} = \frac{8 - (x+1)}{2(x+1)(x-7)} = \frac{7 - x}{2(x+1)(x-7)} = \frac{-1}{2(x+1)}, \ x \neq 7$$

93. False. The range of $f(x)$ is $[-1, \infty)$.

94. True. The first number in each ordered pair corresponds to exactly one second number.

95. $f(x) = \begin{cases} x + 4, & x \le 0 \\ 4 - x^2, & x > 0 \end{cases}$

96. $f(x) = \begin{cases} 1 - x^2, & x \le 0 \\ x + 1, & x > 0 \end{cases}$

97. $f(x) = \begin{cases} 2 - x, & x \le -2 \\ 4, & -2 < x < 3 \\ x + 1, & x \ge 3 \end{cases}$

98. $f(x) = \begin{cases} x^2, & x \le 1 \\ 1, & 1 < x < 4 \\ 5 - x, & x \ge 4 \end{cases}$

99. The domain is the set of inputs of the function and the range is the set of corresponding outputs.

100. An advantage of function notation is that it gives a name to the relationship so it can easily be referenced. When evaluating a function, you see both the input and output values.

101. $12 - \frac{4}{x+2} = \frac{12(x+2) - 4}{x+2} = \frac{12x + 20}{x+2}$

102.
$$\begin{aligned} \frac{3}{x^2 + x - 20} + \frac{x}{x^2 + 4x - 5} &= \frac{3}{(x+5)(x-4)} + \frac{x}{(x+5)(x-1)} \\ &= \frac{3(x-1)}{(x+5)(x-4)(x-1)} + \frac{x(x-4)}{(x+5)(x-1)(x-4)} \\ &= \frac{3x - 3 + x^2 - 4x}{(x+5)(x-4)(x-1)} = \frac{x^2 - x - 3}{(x+5)(x-4)(x-1)} \end{aligned}$$

103. $\dfrac{2x^3 + 11x^2 - 6x}{5x} \cdot \dfrac{x + 10}{2x^2 + 5x - 3} = \dfrac{x(2x^2 + 11x - 6)(x + 10)}{5x(2x - 1)(x + 3)}$

$$= \frac{(2x - 1)(x + 6)(x + 10)}{5(2x - 1)(x + 3)}$$

$$= \frac{(x + 6)(x + 10)}{5(x + 3)}, x \neq 0, \frac{1}{2}$$

104. $\dfrac{x + 7}{2(x - 9)} \div \dfrac{x - 7}{2(x - 9)} = \dfrac{x + 7}{2(x - 9)} \cdot \dfrac{2(x - 9)}{x - 7} = \dfrac{x + 7}{x - 7}, \quad x \neq 9$

Section 1.3 Graphs of Functions

- You should be able to determine the domain and range of a function from its graph.
- You should be able to use the vertical line test for functions.
- You should be able to determine when a function is constant, increasing, or decreasing.
- You should be able to find relative maximum and minimum values of a function.
- You should know that f is
  (a) Odd if $f(-x) = -f(x)$.
  (b) Even if $f(-x) = f(x)$.

Vocabulary Check

1. ordered pairs
2. Vertical Line Test
3. decreasing
4. minimum
5. greatest integer
6. even

1. Domain: All real numbers

   Range: $(-\infty, 1]$

   $f(0) = 1$

2. Domain: all real numbers, $(-\infty, \infty)$

   Range: all real numbers, $(-\infty, \infty)$

   $f(0) = 2$

3. Domain: $[-4, 4]$

   Range: $[0, 4]$

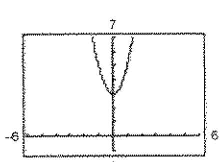
   $f(0) = 4$

4. Domain: all real numbers, $(-\infty, \infty)$

   Range: $[-3, \infty)$

   $f(0) = -3$

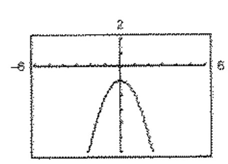
5. $f(x) = 2x^2 + 3$



Domain: All real numbers

Range: $[3, \infty)$

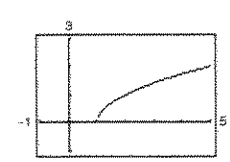
6. $f(x) = -x^2 - 1$



Domain: $(-\infty, \infty)$

Range: $(-\infty, -1]$

7. $f(x) = \sqrt{x - 1}$



Domain: $x - 1 \geq 0 \Rightarrow x \geq 1$ or $[1, \infty)$

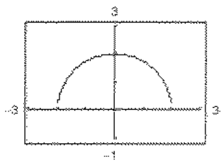
Range: $[0, \infty)$

8. $h(t) = \sqrt{4 - t^2}$

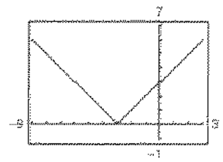
$4 - t^2 \geq 0 \implies t^2 \leq 4$

Domain: $[-2, 2]$

Range: $[0, 2]$



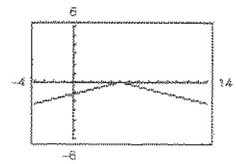
9. $f(x) = |x + 3|$



Domain: All real numbers

Range: $[0, \infty)$

10. $f(x) = -\frac{1}{4}|x - 5|$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 0]$

11. $f(x) = x^2 - x - 6$

(a) Domain: all real numbers

(b) $f(x) = x^2 - x - 6 = (x - 3)(x + 2) = 0 \implies x = 3, -2$

(c) These are the x-intercepts of f.

(d) $f(0) = -6$

(e) This is the y-intercept of f.

(f) $f(1) = 1^2 - 1 - 6 = -6$. The coordinates are $(1, -6)$

(g) $f(-1) = (-1)^2 - (-1) - 6 = -4$. The coordinates are $(-1, -4)$.

(h) $f(-3) = (-3)^2 - (-3) - 6 = 6$. $(-3, f(-3)) = (-3, 6)$.

12. $f(x) = x^3 - 4x$

(a) Domain: all real numbers

(b) $f(x) = x^3 - 4x = x(x^2 - 4) = x(x - 2)(x + 2) = 0 \implies x = 0, 2, -2$

(c) These are the x-intercepts of f.

(d) $f(0) = 0$

(e) This is the y-intercept (and x-intercept) of f.

(f) $f(1) = 1 - 4 = -3$. The coordinates are $(1, -3)$.

(g) $f(-1) = -1 - 4(-1) = 3$. The coordinates are $(-1, 3)$.

(h) $f(-3) = (-3)^3 - 4(-3) = -27 + 12 = -15$. $(-3, f(-3)) = (-3, -15)$.

13. $f(x) = |x - 1| - 2$

(a) Domain: all x

(b) $|x - 1| - 2 = 0 \implies |x - 1| = 2 \implies x = -1, 3$

(c) x-intercepts

(d) $f(0) = |0 - 1| - 2 = -1$

(e) y-intercept

(f) $f(1) = |1 - 1| - 2 = -2$, $(1, -2)$

(g) $f(-1) = |-1 - 1| - 2 = 0$, $(-1, 0)$

(h) $f(-3) = |-3 - 1| - 2 = 2$, $(-3, 2)$

14. $f(x) = \begin{cases} x + 4, & x \le 0 \\ 4 - x^2, & x > 0 \end{cases}$

(a) Domain: all x

(b) $f(x) = 0$ if $x = -4$ or $x = 2$

(c) x-intercepts

(d) $f(0) = 0 + 4 = 4$

(e) y-intercept

(f) $f(1) = 4 - 1^2 = 3, \quad (1, 3)$

(g) $f(-1) = (-1) + 4 = 3, \quad (-1, 3)$

(h) $f(-3) = -3 + 4 = 1, \quad (-3, 1)$

15. $y = \frac{1}{2}x^2$

A vertical line intersects the graph just once, so y is a function of x. Graph $y_1 = \frac{1}{2}x^2$.

16. $x - y^2 = 1 \implies y = \pm\sqrt{x - 1}$

y is not a function of x. The vertical line $x = 2$ intersects the graph twice. Graph $y_1 = \sqrt{x - 1}$ and $y_2 = -\sqrt{x - 1}$.

17. $x^2 + y^2 = 25$

A vertical line intersects the graph more than once, so y is not a function of x. Graph the circle as

$y_1 = \sqrt{25 - x^2}$

$y_2 = -\sqrt{25 - x^2}$.

18. $x^2 = 2xy - 1$

A vertical line intersects the graph just once, so y is a function of x. Solve for y and graph

$$y = \frac{x^2 + 1}{2x}.$$

19. $f(x) = \dfrac{3}{2}x$

f is increasing on $(-\infty, \infty)$.

20. $f(x) = x^2 - 4x$

The graph is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

21. $f(x) = x^3 - 3x^2 + 2$

f is increasing on $(-\infty, 0)$ and $(2, \infty)$.

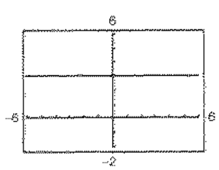
f is decreasing on $(0, 2)$.

22. $f(x) = \sqrt{x^2 - 1}$

The graph is decreasing on $(-\infty, -1)$ and increasing on $(1, \infty)$.

23. $f(x) = 3$

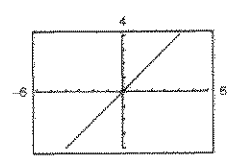
(a)



(b) f is constant on $(-\infty, \infty)$.

24. $f(x) = x$

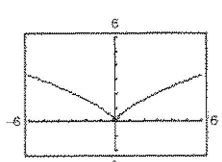
(a)



(b) The graph is increasing on $(-\infty, \infty)$.

25. $f(x) = x^{2/3}$

(a)

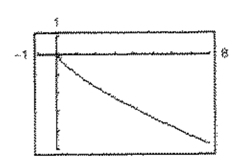


(b) Increasing on $(0, \infty)$

Decreasing on $(-\infty, 0)$

26. $f(x) = -x^{3/4}$

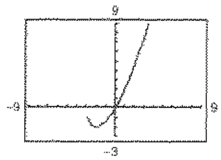
(a)



(b) The graph is decreasing on $(0, \infty)$.

27. $f(x) = x\sqrt{x + 3}$

(a)

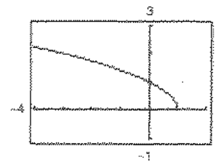


(b) Increasing on $(-2, \infty)$

Decreasing on $(-3, -2)$

28. $f(x) = \sqrt{1 - x}$

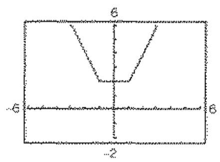
(a)



(b) f is decreasing on $(-\infty, 1)$.

29. $f(x) = |x + 1| + |x - 1|$

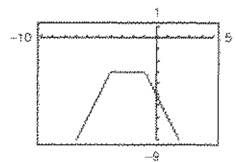
(a)



(b) Increasing on $(1, \infty)$, constant on $(-1, 1)$, decreasing on $(-\infty, -1)$

30. $f(x) = -|x + 4| - |x + 1|$

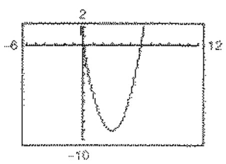
(a)



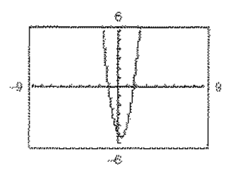
(b) The graph is increasing on $(-\infty, -4)$, constant on $(-4, -1)$, and decreasing on $(-1, \infty)$.

31. $f(x) = x^2 - 6x$

Relative minimum: $(3, -9)$



32. $f(x) = 3x^2 - 2x - 5$

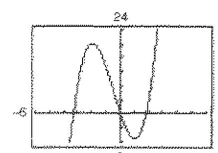


Relative minimum: $(0.33, -5.33)$

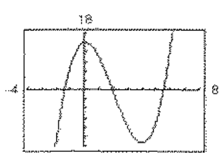
33. $y = 2x^3 + 3x^2 - 12x$

Relative minimum: $(1, -7)$

Relative maximum: $(-2, 20)$



34. $y = x^3 - 6x^2 + 15$

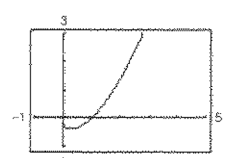


Relative minimum: $(4, -17)$

Relative maximum: $(0, 15)$

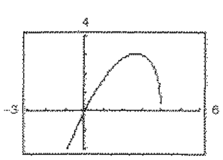
35. $h(x) = (x - 1)\sqrt{x}$

Relative minimum: $(0.33, -0.38)$



$(0, 0)$ is not a relative maximum because it occurs at the endpoint of the domain $[0, \infty)$.

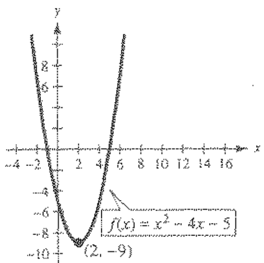
36. $g(x) = x\sqrt{4 - x}$



Maximum: $(2.67, 3.08)$

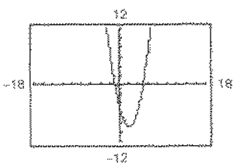
37. $f(x) = x^2 - 4x - 5$

(a)



Minimum: $(2, -9)$

(b)

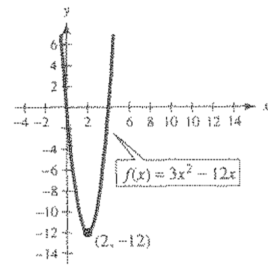


Minimum: $(2, -9)$

(c) Answers are the same.

38. $f(x) = 3x^2 - 12x$

(a)



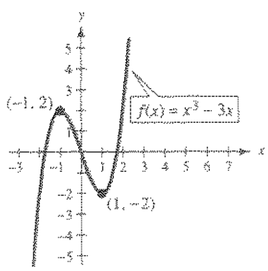
Relative minimum: $(2, -12)$

(b) Relative minimum: $(2, -12)$

(c) Answers are the same.

39. $f(x) = x^3 - 3x$

(a)



Relative maximum: $(-1, 2)$

Relative minimum: $(1, -2)$

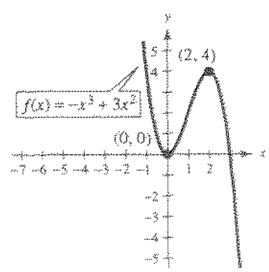
(b) Relative maximum: $(-1, 2)$

Relative minimum: $(1, -2)$

(c) Answers are the same.

40. $f(x) = -x^3 + 3x^2$

(a)



Relative maximum: $(2, 4)$

Relative minimum: $(0, 0)$

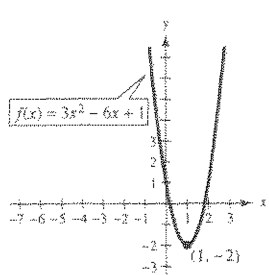
(b) Relative maximum: $(2, 4)$

Relative minimum: $(0, 0)$

(c) Answers are the same.

41. $f(x) = 3x^2 - 6x + 1$

(a)



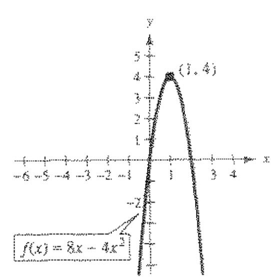
Relative minimum: $(1, -2)$

(b) Relative minimum: $(1, -2)$

(c) Answers are the same.

42. $f(x) = 8x - 4x^2$

(a)

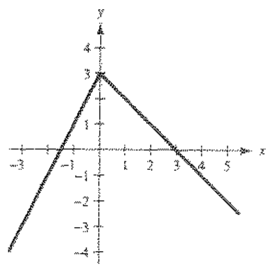


Relative maximum: $(1, 4)$

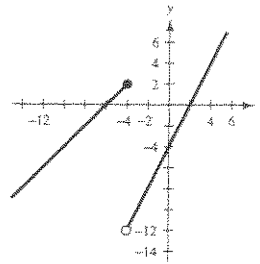
(b) Relative maximum: $(1, 4)$

(c) Answers are the same.

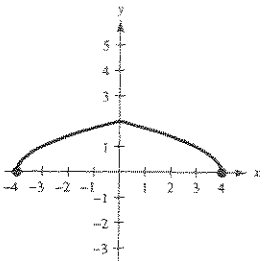
43. $f(x) = \begin{cases} 2x + 3, & x < 0 \\ 3 - x, & x \geq 0 \end{cases}$



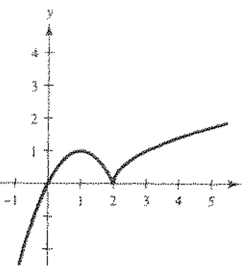
44. $f(x) = \begin{cases} x + 6, & x \leq -4 \\ 2x - 4, & x > -4 \end{cases}$



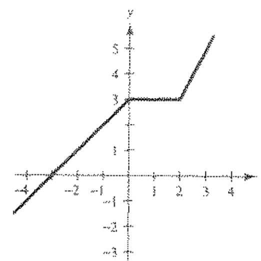
45. $f(x) = \begin{cases} \sqrt{x + 4}, & x < 0 \\ \sqrt{4 - x}, & x \geq 0 \end{cases}$



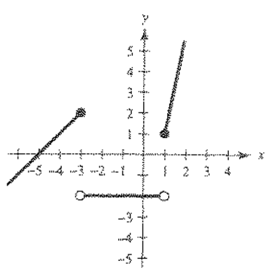
46. $f(x) = \begin{cases} 1 - (x - 1)^2, & x \leq 2 \\ \sqrt{x - 2}, & x > 2 \end{cases}$



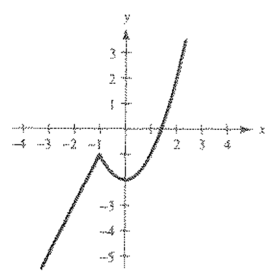
47. $f(x) = \begin{cases} x + 3, & x \leq 0 \\ 3, & 0 < x \leq 2 \\ 2x - 1, & x > 2 \end{cases}$



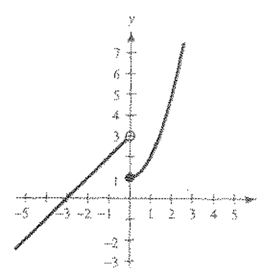
48. $g(x) = \begin{cases} x + 5, & x \leq -3 \\ -2, & -3 < x < 1 \\ 5x - 4, & x \geq 1 \end{cases}$



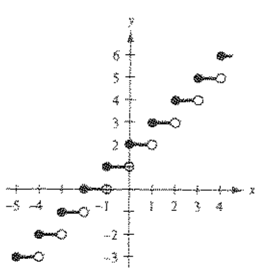
49. $f(x) = \begin{cases} 2x + 1, & x \leq -1 \\ x^2 - 2, & x > -1 \end{cases}$



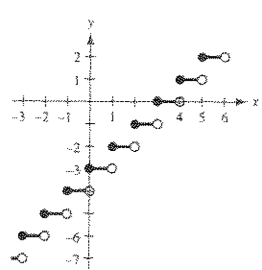
50. $h(x) = \begin{cases} 3 + x, & x < 0 \\ x^2 + 1, & x \geq 0 \end{cases}$



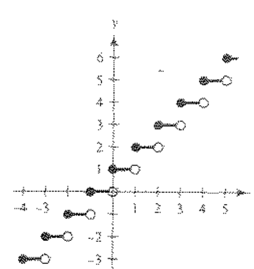
51. $f(x) = [\![x]\!] + 2$



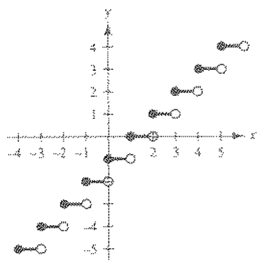
52. $f(x) = [\![x]\!] - 3$



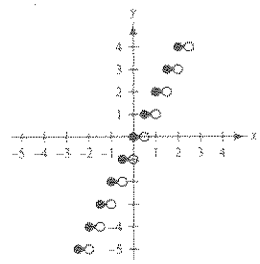
53. $f(x) = [\![x - 1]\!] + 2$



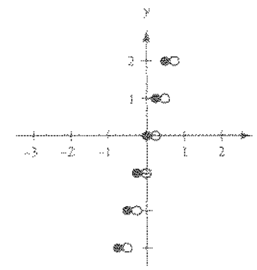
54. $f(x) = [\![x - 2]\!] + 1$



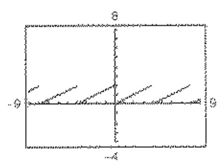
55. $f(x) = [\![2x]\!]$



56. $f(x) = [\![4x]\!]$



57. $s(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)$

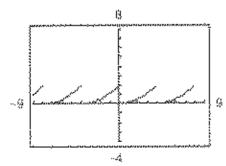


Domain: $(-\infty, \infty)$

Range: $[0, 2)$

Sawtooth pattern

58. $g(x) = 2\left(\frac{1}{4}x - [\![\frac{1}{4}x]\!]\right)^2$



Domain: $(-\infty, \infty)$

Range: $[0, 2)$

Pattern: Sawtooth

59. $f(-t) = (-t)^2 + 2(-t) - 3$

$= t^2 - 2t - 3$

$\neq f(t) \neq -f(t)$

f is neither even nor odd.

60. $f(-x) = (-x)^6 - 2(-x)^2 + 3$

$= x^6 - 2x^2 + 3 = f(x).$

f is even.

61. $g(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -g(x)$

g is odd.

62. $h(x) = x^3 - 5$

$h(-x) = (-x)^3 - 5$

$= -x^3 - 5$

$\neq h(x)$

$\neq -h(x)$

The function is neither odd nor even.

63. $f(-x) = (-x)\sqrt{1 - (-x)^2}$

$= -x\sqrt{1 - x^2}$

$= -f(x)$

The function is odd.

64. $f(-x) = (-x)\sqrt{(-x) + 5}$

$= -x\sqrt{-x + 5}$

$\neq f(x)$

$\neq -f(x)$

The function is neither even nor odd.

65. $g(-s) = 4(-s)^{2/3}$

$= 4s^{2/3}$

$= g(s)$

The function is even.

66. Because the domain is $s \geq 0$, the function is neither even nor odd.

67. $\left(-\frac{3}{2}, 4\right)$

(a) If f is even, another point is $\left(\frac{3}{2}, 4\right)$.

(b) If f is odd, another point is $\left(\frac{3}{2}, -4\right)$.

68. $\left(-\frac{5}{3}, -7\right)$

(a) If f is even, another point is $\left(\frac{5}{3}, -7\right)$.

(b) If f is odd, another point is $\left(\frac{5}{3}, 7\right)$.

69. $(4, 9)$

(a) If f is even, another point is $(-4, 9)$.

(b) If f is odd, another point is $(-4, -9)$.

70. $(5, -1)$

(a) If f is even, another point is $(-5, -1)$.

(b) If f is odd, another point is $(-5, 1)$.

71. $(x, -y)$

(a) If f is even, another point is $(-x, -y)$.

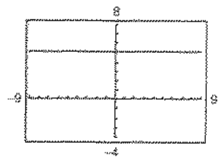
(b) If f is odd, another point is $(-x, y)$.

72. $(2a, 2c)$

(a) If f is even, another point is $(-2a, 2c)$.

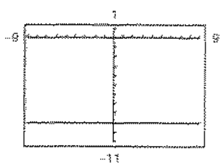
(b) If f is odd, another point is $(-2a, -2c)$.

73. $f(x) = 5$, even

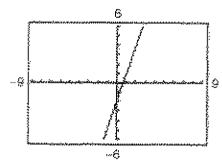


74. $f(x) = -9$

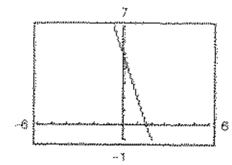
f is even.



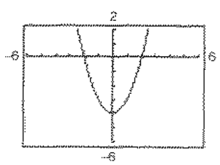
75. $f(x) = 3x - 2$ is neither even nor odd.



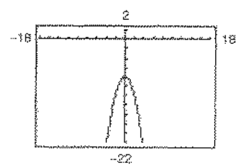
76. $f(x) = 5 - 3x$ is neither even nor odd.



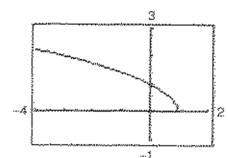
77. $h(x) = x^2 - 4$, even



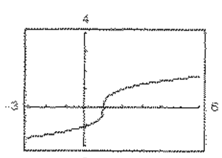
78. $f(x) = -x^2 - 8$ is even.



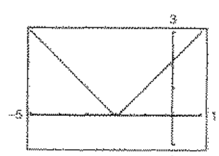
79. $f(x) = \sqrt{1 - x}$ is neither even nor odd.



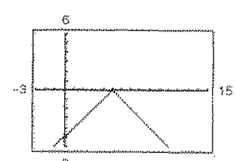
80. $g(t) = \sqrt[3]{t - 1}$ is neither even nor odd.



81. $f(x) = |x + 2|$ is neither even nor odd.



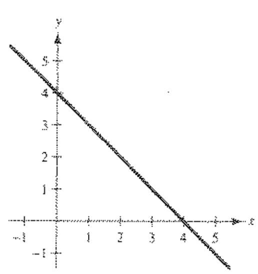
82. $f(x) = -|x - 5|$ is neither even nor odd.



83. $f(x) = 4 - x \geq 0$

$4 \geq x$

$(-\infty, 4]$



84. $f(x) = 4x + 2$

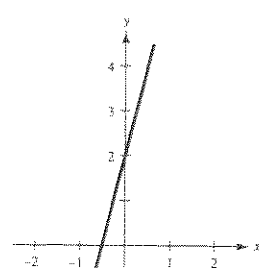
$f(x) \geq 0$

$4x + 2 \geq 0$

$4x \geq -2$

$x \geq -\frac{1}{2}$

$\left[-\frac{1}{2}, \infty\right)$

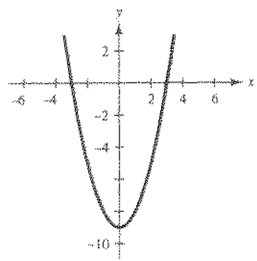


85. $f(x) = x^2 - 9 \geq 0$

$x^2 \geq 9$

$x \geq 3$ or $x \leq -3$

$[3, \infty)$ or $(-\infty, -3]$



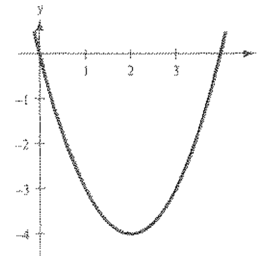
86. $f(x) = x^2 - 4x$

$f(x) \geq 0$

$x^2 - 4x \geq 0$

$x(x - 4) \geq 0$

$(-\infty, 0], [4, \infty)$

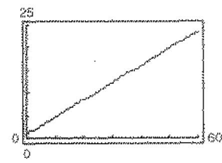


87. (a) The second model is correct. For instance,

$$C_2\left(\tfrac{1}{2}\right) = 1.05 - 0.38[[-(\tfrac{1}{2} - 1)]]$$

$$= 1.05 - 0.38[[\tfrac{1}{2}]] = 1.05.$$

(b)



The cost of an 18-minute 45-second call is

$$C_2\left(18\tfrac{45}{60}\right) = C_2(18.75) = 1.05 - 0.38[[-(18.75 - 1)]]$$

$$= 1.05 - 0.38[[-17.75]] = 1.05 - 0.38(-18)$$

$$= 1.05 + 0.38(18) = \$7.89.$$

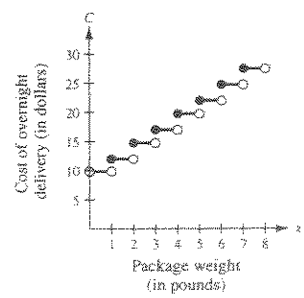
88. *Model*: (Total cost) = (Flat rate) + (Rate per pound)

Labels: Total cost $= C$

Flat rate $= 9.80$

Rate per pound $= 2.50[[x]],\ x > 0$

Equation: $C = 9.80 + 2.50[[x]],\ x > 0$



89. $h =$ top $-$ bottom

$= (-x^2 + 4x - 1) - 2$

$= -x^2 + 4x - 3,\ 1 \leq x \leq 3$

90. $h =$ top $-$ bottom

$= 3 - (4x - x^2)$

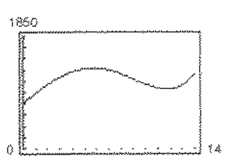
$= 3 - 4x + x^2,$

$0 \leq x \leq 1$

91. $P(t) = 0.0108t^4 - 0.211t^3 + 0.40t^2 + 7.9t + 1791$

$0 \leq t \leq 14$

(a)



(b) P is increasing from 1990 ($t = 0$) to 1995 ($t \approx 5.7$), and from 2001 ($t \approx 11.8$) to 2004. P is decreasing from 1995 to 2001.

(c) The maximum population was about 1,821,000 in 1995 ($t \approx 5.7$).

92.

Interval	*Intake Pipe*	*Drainpipe 1*	*Drainpipe 2*
[0, 5]	Open	Closed	Closed
[5, 10]	Open	Open	Closed
[10, 20]	Closed	Closed	Closed
[20, 30]	Closed	Closed	Open
[30, 40]	Open	Open	Open
[40, 45]	Open	Closed	Open
[45, 50]	Open	Open	Open
[50, 60]	Open	Open	Closed

93. False. The domain of $f(x) = \sqrt{x^2}$ is the set of all real numbers.

94. False. The domain must be symmetric about the y-axis.

95. c **96.** d **97.** b **98.** e **99.** a **100.** f

101.
$$f(x) = a_{2n+1}x^{2n+1} + a_{2n-1}x^{2n-1} + \cdots + a_3x^3 + a_1x$$
$$f(-x) = a_{2n+1}(-x)^{2n+1} + a_{2n-1}(-x)^{2n-1} + \cdots + a_3(-x)^3 + a_1(-x)$$
$$= -a_{2n+1}x^{2n+1} - a_{2n-1}x^{2n-1} - \cdots - a_3x^3 - a_1x = -f(x)$$
Therefore, $f(x)$ is odd.

102.
$$f(x) = a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0$$
$$f(-x) = a_{2n}(-x)^{2n} + a_{2n-2}(-x)^{2n-2} + \cdots + a_2(-x)^2 + a_0$$
$$= a_{2n}x^{2n} + a_{2n-2}x^{2n-2} + \cdots + a_2x^2 + a_0 = f(x)$$
$f(-x) = f(x)$; thus, $f(x)$ is even.

103. f is an even function.

(a) $g(x) = -f(x)$ is even because $g(-x) = -f(-x) = -f(x) = g(x)$.

(b) $g(x) = f(-x)$ is even because $g(-x) = f(-(-x)) = f(x) = f(-x) = g(x)$.

(c) $g(x) = f(x) - 2$ is even because $g(-x) = f(-x) - 2 = f(x) - 2 = g(x)$.

(d) $g(x) = -f(x - 2)$ is neither even nor odd because $g(-x) = -f(-x - 2) = -f(x + 2) \neq g(x)$ nor $-g(x)$.

104. Yes, $x = y^2 + 1$ defines x as a function of y. (But not y as a function of x)

105. No, $x^2 + y^2 = 25$ does not represent x as a function of y. For instance, $(-3, 4)$ and $(3, 4)$ both lie on the graph.

106. Answers will vary.

107. $-2x^2 + 8x$

Terms: $-2x^2, 8x$

Coefficients: $-2, 8$

108. Terms: $3x$, 10

Coefficient: 3

109. $\frac{x}{3} - 5x^2 + x^3$

Terms: $\frac{x}{3}, -5x^2, x^3$

Coefficients: $\frac{1}{3}, -5, 1$

110. Terms: $7x^4$, $\sqrt{2}x^2$

Coefficient: 7, $\sqrt{2}$

111. (a) $d = \sqrt{(6 - (-2))^2 + (3 - 7)^2}$

$= \sqrt{64 + 16} = \sqrt{80} = 4\sqrt{5}$

(b) midpoint $= \left(\frac{-2 + 6}{2}, \frac{7 + 3}{2}\right) = (2, 5)$

112. (a) $d = \sqrt{(-5 - 3)^2 + (0 - 6)^2} = \sqrt{64 + 36} = \sqrt{100} = 10$

(b) midpoint $= \left(\frac{-5 + 3}{2}, \frac{0 + 6}{2}\right) = (-1, 3)$

113. (a) $d = \sqrt{\left(-\frac{3}{2} - \frac{5}{2}\right)^2 + (4 - (-1))^2} = \sqrt{16 + 25} = \sqrt{41}$

(b) midpoint $= \left(\frac{\frac{5}{2} - \frac{3}{2}}{2}, \frac{-1 + 4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

114. (a) $d = \sqrt{\left(-6 - \frac{3}{4}\right)^2 + \left(\frac{2}{3} - \frac{1}{6}\right)^2} = \sqrt{\left(\frac{-27}{4}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{\sqrt{733}}{4}$

(b) midpoint $= \left(\frac{-6 + \frac{3}{4}}{2}, \frac{\frac{2}{3} + \frac{1}{6}}{2}\right) = \left(\frac{-21}{8}, \frac{5}{12}\right)$

115. $f(x) = 5x - 1$

(a) $f(6) = 5(6) - 1 = 29$

(b) $f(-1) = 5(-1) - 1 = -6$

(c) $f(x - 3) = 5(x - 3) - 1 = 5x - 16$

116. $f(x) = -x^2 - x + 3$

(a) $f(4) = -(4)^2 - 4 + 3 = -17$

(b) $f(-2) = -(-2)^2 - (-2) + 3 = 1$

(c) $f(x - 2) = -(x - 2)^2 - (x - 2) + 3$

$= -(x^2 - 4x + 4) - x + 2 + 3$

$= -x^2 + 3x + 1$

117. $f(x) = x\sqrt{x - 3}$

(a) $f(3) = 3\sqrt{3 - 3} = 0$

(b) $f(12) = 12\sqrt{12 - 3}$

$= 12\sqrt{9} = 12(3) = 36$

(c) $f(6) = 6\sqrt{6 - 3} = 6\sqrt{3}$

118. $f(x) = -\frac{1}{2}x|x + 1|$

(a) $f(-4) = -\frac{1}{2}(-4)|-4 + 1| = 2(3) = 6$

(b) $f(10) = -\frac{1}{2}(10)|10 + 1| = -5(11) = -55$

(c) $f\left(-\frac{2}{3}\right) = -\frac{1}{2}\left(-\frac{2}{3}\right)\left|-\frac{2}{3} + 1\right| = \frac{1}{3}\left(\frac{1}{3}\right) = \frac{1}{9}$

119. $f(x) = x^2 - 2x + 9$

$f(3 + h) = (3 + h)^2 - 2(3 + h) + 9 = 9 + 6h + h^2 - 6 - 2h + 9$

$= h^2 + 4h + 12$

$f(3) = 3^2 - 2(3) + 9 = 12$

$$\frac{f(3 + h) - f(3)}{h} = \frac{(h^2 + 4h + 12) - 12}{h} = \frac{h(h + 4)}{h} = h + 4, h \neq 0$$

120. $f(x) = 5 + 6x - x^2$

$$f(6 + h) = 5 + 6(6 + h) - (6 + h)^2 = 5 + 36 + 6h - (36 + 12h + h^2) = -h^2 - 6h + 5$$

$$f(6) = 5 + 6(6) - 6^2 = 5$$

$$\frac{f(h + 6) - f(6)}{h} = \frac{(-h^2 - 6h + 5) - 5}{h} = \frac{h(-h - 6)}{h} = -h - 6, h \neq 0$$

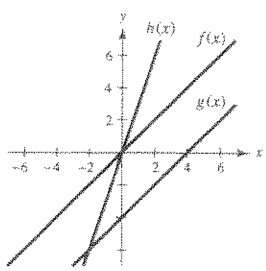
Section 1.4 Shifting, Reflecting, and Stretching Graphs

- You should know the graphs of the most commonly used functions in algebra, and be able to reproduce them on your graphing utility.
  - (a) Constant function: $f(x) = c$
  - (b) Identity function: $f(x) = x$
  - (c) Absolute value function: $f(x) = |x|$
  - (d) Square root function: $f(x) = \sqrt{x}$
  - (e) Squaring function: $f(x) = x^2$
  - (f) Cubing function: $f(x) = x^3$
- You should know how the graph of a function is changed by vertical and horizontal shifts.
- You should know how the graph of a function is changed by reflection.
- You should know how the graph of a function is changed by nonrigid transformations, like stretches and shrinks.
- You should know how the graph of a function is changed by a sequence of transformations.

Vocabulary Check

1. quadratic function
2. absolute value function
3. rigid transformations
4. $-f(x), f(-x)$
5. $c > 1, 0 < c < 1$
6. (a) ii (b) iv (c) iii (d) i

1.



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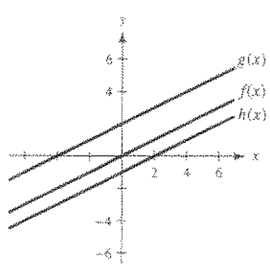

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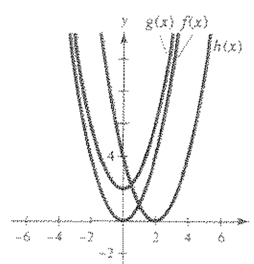

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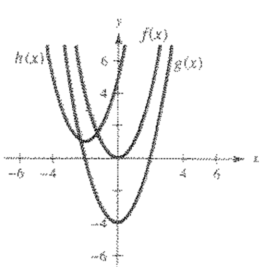

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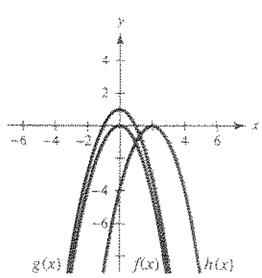

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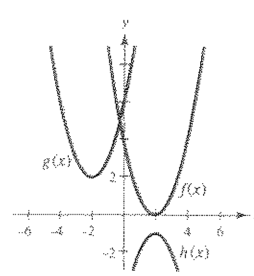

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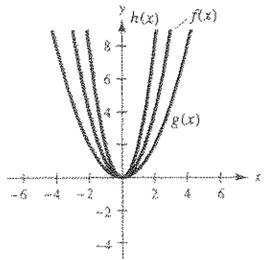

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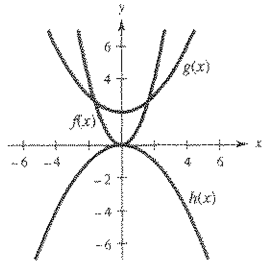

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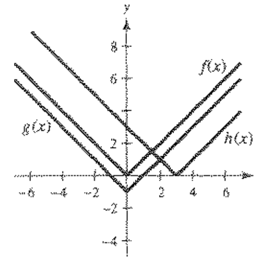

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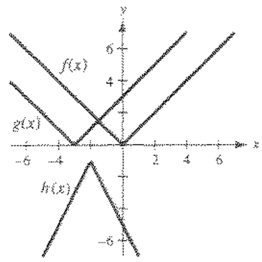

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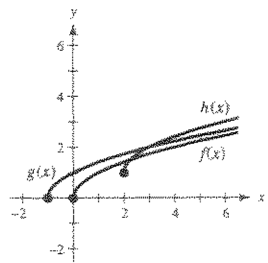

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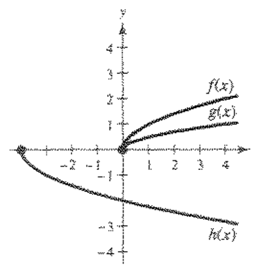
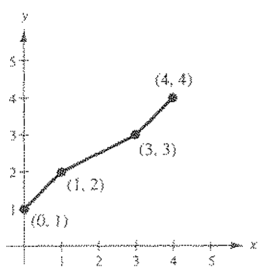
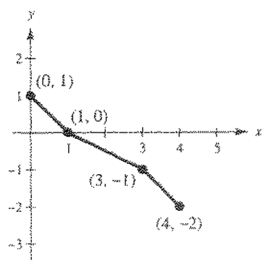


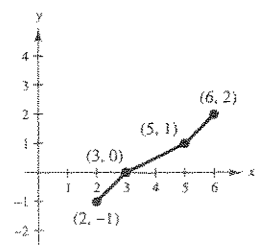
13. (a) $y = f(x) + 2$



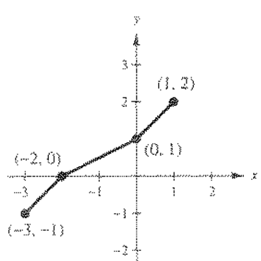
(b) $y = -f(x)$



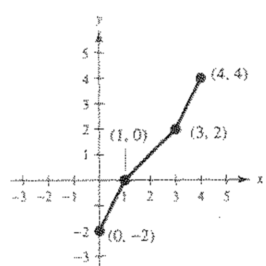
(c) $y = f(x - 2)$



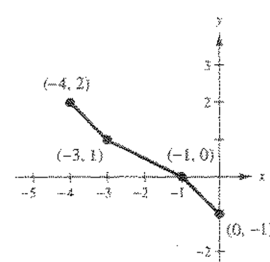
(d) $y = f(x + 3)$



(e) $y = 2f(x)$

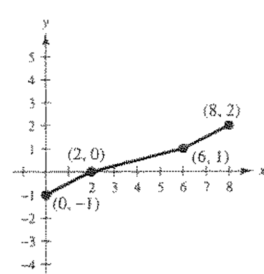


(f) $y = f(-x)$

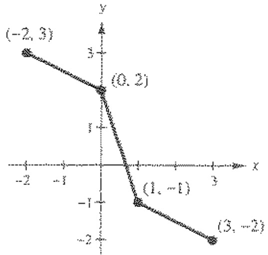


(g) Let $g(x) = f\left(\frac{1}{2}x\right)$. Then from the graph,

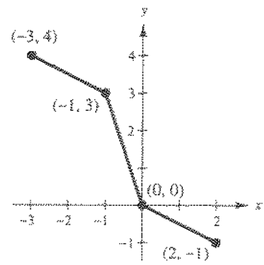
$$g(0) = f\left(\tfrac{1}{2}(0)\right) = f(0) = -1$$
$$g(2) = f\left(\tfrac{1}{2}(2)\right) = f(1) = 0$$
$$g(6) = f\left(\tfrac{1}{2}(6)\right) = f(3) = 1$$
$$g(8) = f\left(\tfrac{1}{2}(8)\right) = f(4) = 2$$



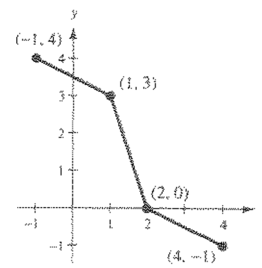
14. (a)



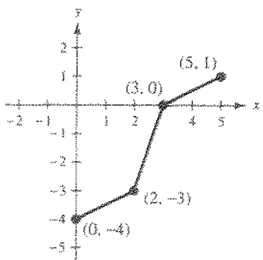
(b)



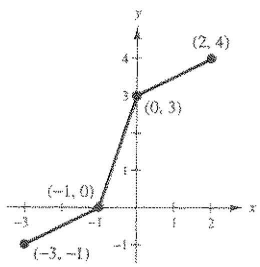
(c)



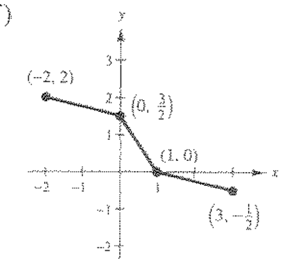
(d)



(e)

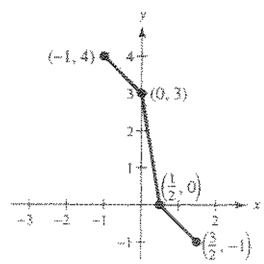


(f)



(g) Let $g(x) = f(2x)$. Then from the graph,

$$g(-1) = f(2(-1)) = f(-2) = 4$$
$$g(0) = f(2(0)) = f(0) = 3$$
$$g\left(\tfrac{1}{2}\right) = f\left(2\left(\tfrac{1}{2}\right)\right) = f(1) = 0$$
$$g\left(\tfrac{3}{2}\right) = f\left(2\left(\tfrac{3}{2}\right)\right) = f(3) = -1$$



15. Horizontal shift three units to left of $y = x$: $y = x + 3$ (or vertical shift three units upward)

16. Constant function: $y = 7$

17. Vertical shift one unit downward of $y = x^2$

$y = x^2 - 1$

18. Horizontal shift of $y = |x|$: $y = |x + 2|$

19. Reflection in the x-axis and a vertical shift one unit upward of $y = \sqrt{x}$: $y = 1 - \sqrt{x}$

20. Reflection in the x-axis and a vertical shift one unit upward of $y = x^3$: $y = 1 - x^3$

21. $y = -\sqrt{x} - 1$ is $f(x)$ reflected in the x-axis, followed by a vertical shift one unit downward.

22. $y = \sqrt{x} + 2$ is $f(x) = \sqrt{x}$ shifted vertically upwards two units.

23. $y = \sqrt{x - 2}$ is $f(x)$ shifted right two units.

24. $y = \sqrt{x + 4}$ is $f(x)$ shifted left four units.

25. $y = 2\sqrt{x}$ is a vertical stretch of $f(x) = \sqrt{x}$.

26. $y = \sqrt{-x + 3}$ is $f(x)$ reflected in the y-axis, followed by a horizontal shift to the right three units.

27. $y = |x + 5|$ is $f(x)$ shifted left five units.

28. $y = |x| - 3$ is $f(x) = |x|$ shifted down three units.

29. $y = -|x|$ is $f(x)$ reflected in the x-axis.

30. $y = |-x|$ is a reflection in the y-axis. In fact $y = |-x| = |x|$.

31. $y = 4|x|$ is a vertical stretch of $f(x)$.

32. $y = \left|\frac{1}{2}x\right| = \frac{1}{2}|x|$ is a vertical shrink.

33. $g(x) = 4 - x^3$ is obtained from $f(x)$ by a reflection in the x-axis followed by a vertical shift upward of four units.

34. $g(x) = -(x - 1)^3$ is obtained by a horizontal shift of one unit to the right, followed by a reflection in the x-axis.

35. $h(x) = \frac{1}{4}(x + 2)^3$ is obtained from $f(x)$ by a left shift of two units and a vertical shrink by a factor of $\frac{1}{4}$.

36. $h(x) = -2(x - 1)^3 + 3$ is obtained from $f(x)$ by a right shift of one unit, a vertical stretch by a factor of two, a reflection in the x-axis, and a vertical shift three units upward.

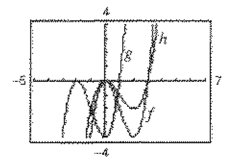
37. $p(x) = \left(\frac{1}{3}x\right)^3 + 2$ is obtained from $f(x)$ by a horizontal stretch followed by a vertical shift two units upward.

38. $p(x) = [3(x - 2)]^3$ is obtained from $f(x)$ by a right shift of two units, followed by a vertical stretch.

39. $f(x) = x^3 - 3x^2$

$g(x) = f(x + 2) = (x + 2)^3 - 3(x + 2)^2$ is a horizontal shift two units to left.

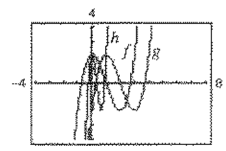
$h(x) = \frac{1}{2}f(x) = \frac{1}{2}(x^3 - 3x^2)$ is a vertical shrink.



40. $f(x) = x^3 - 3x^2 + 2$

$g(x) = f(x - 1) = (x - 1)^3 - 3(x - 1)^2 + 2$ is a horizontal shift one unit to the right.

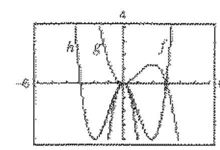
$h(x) = f(3x) = (3x)^3 - 3(3x)^2 + 2$ is a horizontal shrink.



41. $f(x) = x^3 - 3x^2$

$g(x) = -\frac{1}{3}f(x) = -\frac{1}{3}(x^3 - 3x^2)$ reflection in the x-axis and vertical shrink

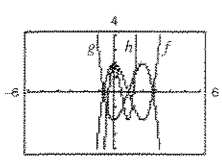
$h(x) = f(-x) = (-x)^3 - 3(-x)^2$ reflection in the y-axis



42. $f(x) = x^3 - 3x^2 + 2$

$g(x) = -f(x) = -(x^3 - 3x^2 + 2)$ is a reflection in the x-axis.

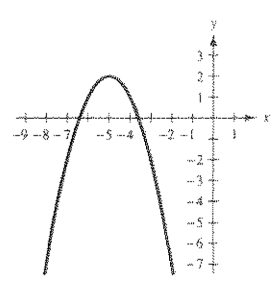
$h(x) = f(2x) = (2x)^3 - 3(2x)^2 + 2$ is a horizontal shrink.



43. (a) $f(x) = x^2$

(b) $g(x) = 2 - (x + 5)^2$ is obtained from f by a horizontal shift to the left five units, a reflection in the x-axis, and a vertical shift upward two units.

(c)

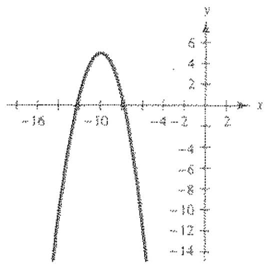


(d) $g(x) = 2 - f(x + 5)$

44. (a) $f(x) = x^2$

(b) $g(x) = -(x + 10)^2 + 5$ is obtained from f by a horizontal shift 10 units to the left, a reflection in the x-axis, and a vertical shift 5 units upward.

(c)

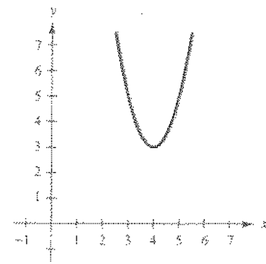


(d) $g(x) = -f(x + 10) + 5$

45. (a) $f(x) = x^2$

(b) $g(x) = 3 + 2(x - 4)^2$ is obtained from f by a horizontal shift four units to the right, a vertical stretch of 2, and a vertical shift upward three units.

(c)

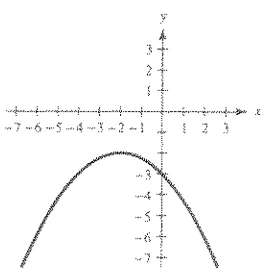


(d) $g(x) = 3 + 2f(x - 4)$

46. (a) $f(x) = x^2$

(b) $g(x) = -\frac{1}{4}(x + 2)^2 - 2$ is obtained from f by a horizontal shift two units to the left, a vertical shrink of $\frac{1}{4}$, a reflection in the x-axis, and a vertical shift two units downward.

(c)

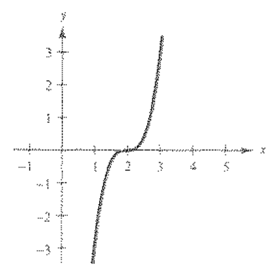


(d) $g(x) = -\frac{1}{4}f(x + 2) - 2$

47. (a) $f(x) = x^3$

(b) $g(x) = 3(x - 2)^3$ is obtained from f by a horizontal shift two units to the right followed by a vertical stretch of 3.

(c)

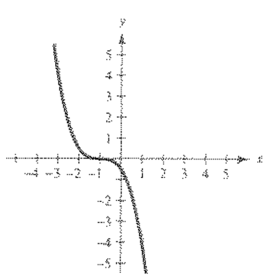


(d) $g(x) = 3f(x - 2)$

48. (a) $f(x) = x^3$

(b) $g(x) = -\frac{1}{2}(x + 1)^3$ is obtained from f by a horizontal shift one unit to the left, a vertical shrink, and a reflection in the x-axis.

(c)

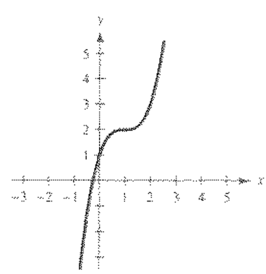


(d) $g(x) = -\frac{1}{2}f(x + 1)$

49. (a) $f(x) = x^3$

(b) $g(x) = (x - 1)^3 + 2$ is obtained from f by a horizontal shift one unit to the right, and a vertical shift upward two units.

(c)

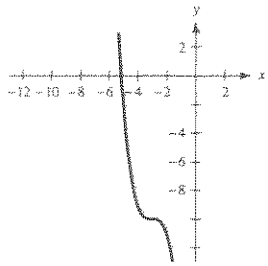


(d) $g(x) = f(x - 1) + 2$

50. (a) $f(x) = x^3$

(b) $g(x) = -(x + 3)^3 - 10$ is obtained from f by a horizontal shift 3 units to the left, a reflection in the x-axis, and a vertical shift 10 units downward.

(c)

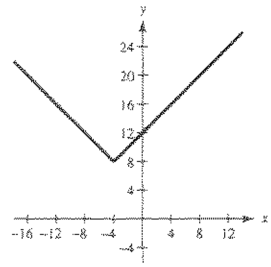


(d) $g(x) = -f(x + 3) - 10$

51. (a) $f(x) = |x|$

(b) $g(x) = |x + 4| + 8$ is obtained from f by a horizontal shift four units to the left, followed by a vertical shift eight units upward.

(c)

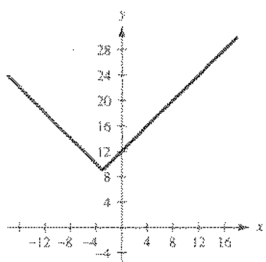


(d) $g(x) = f(x + 4) + 8$

52. (a) $f(x) = |x|$

(b) $g(x) = |x + 3| + 9$ is obtained from f by a horizontal shift three units to the left, followed by a vertical shift nine units upward.

(c)

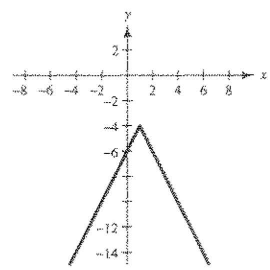


(d) $g(x) = f(x + 3) + 9$

53. (a) $f(x) = |x|$

(b) $g(x) = -2|x - 1| - 4$ is obtained from f by a horizontal shift one unit to the right, a vertical stretch of 2, a reflection in the x-axis, and a vertical shift downward four units.

(c)

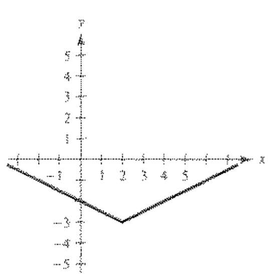


(d) $g(x) = -2f(x - 1) - 4$

54. (a) $f(x) = |x|$

(b) $g(x) = \frac{1}{2}|x - 2| - 3$ is obtained from f by a horizontal shift two units to the right, a vertical shrink, and a vertical shift three units downward.

(c)

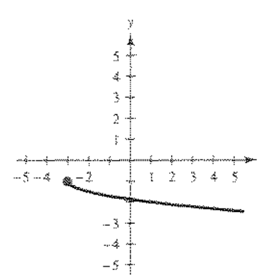


(d) $g(x) = \frac{1}{2}f(x - 2) - 3$

55. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -\frac{1}{2}\sqrt{x + 3} - 1$ is obtained from f by a horizontal shift three units to the left, a vertical shrink, a reflection in the x-axis, and a vertical shift one unit downward.

(c)

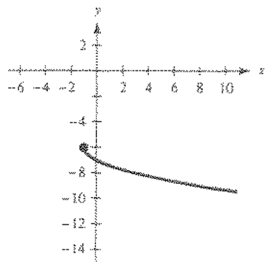


(d) $g(x) = -\frac{1}{2}f(x + 3) - 1$

56. (a) $f(x) = \sqrt{x}$

(b) $g(x) = -\sqrt{x+1} - 6$ is obtained from f by a horizontal shift one unit to the left, a reflection in the x-axis, and a vertical shift six units downward.

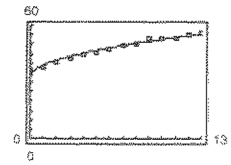
(c)



(d) $g(x) = -f(x+1) - 6$

57. (a) $F(t) = 33.0 + 6.2\sqrt{t}$ is a vertical stretch of $f(t) = \sqrt{t}$, followed by a vertical shift of 33.0.

(b)



(c) $G(t) = F(t+13) = 33.0 + 6.2\sqrt{t+13}$,

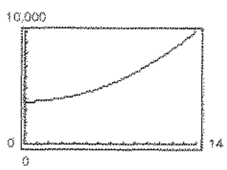
$-13 \le t \le 0$.

$G(-13) = F(0)$ corresponds to 1990.

$G(0) = F(13)$ corresponds to 2003.

58. (a) $M(t) = 32.3t^2 + 3769$ is a vertical stretch of $f(t) = t^2$ by 32.3, followed by a vertical shift of 3769.

(b)



(c) $M(t) = 32.3t^2 + 3769 > 10{,}000$

$$32.3t^2 > 6231$$

$$t^2 > 192.91$$

$$t > 13.9$$

The debt will exceed 10 trillion dollars in 2003.

(d) $G(t) = M(t+10) = 32.3(t+10)^2 + 3769, \quad -10 \le t \le 4$

$G(0) = M(10)$ corresponds to 2000.

$G(-10) = M(0)$ corresponds to 1990.

59. False. $y = f(-x)$ is a reflection in the y-axis.

60. False. $y = -f(x)$ is a reflection in the x-axis.

61. (a) $y = f(-x)$ is a reflection in the y-axis, so the x-intercepts are $x = -2$ and $x = 3$.

(b) $y = -f(x)$ is a reflection in the x-axis, so the x-intercepts are $x = 2$ and $x = -3$.

(c) $y = 2f(x)$ is a vertical stretch, so the x-intercepts are the same: $x = 2, -3$.

(d) $y = f(x) + 2$ is a vertical shift, so you cannot determine the x-intercepts.

(e) $y = f(x-3)$ is a horizontal shift 3 units to the right, so the x-intercepts are $x = 5$ and $x = 0$.

62. (a) $y = f(-x)$ is a reflection in the y-axis, so the x-intercepts are $x = 1$ and $x = -4$.

(b) $y = -f(x)$ is a reflection in the x-axis, so the x-intercepts are the same $x = -1, 4$.

(c) $y = 2f(x)$ is a vertical stretch, so the x-intercepts are the same: $x = -1, 4$.

(d) $y = f(x) - 1$ is a vertical shift, so you cannot determine the x-intercepts.

(e) $y = f(x-2)$ is a horizontal shift 2 units to the right, so the x-intercepts are $x = 1$ and $x = 6$.

63. (a) $y = f(-x)$ is a reflection in the y-axis, so the graph is increasing on $(-\infty, -2)$ and decreasing on $(-2, \infty)$.

(b) $y = -f(x)$ is a reflection in the x-axis, so the graph is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

(c) $y = 2f(x)$ is a vertical stretch, so the graph is increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$.

(d) $y = f(x) - 3$ is a vertical shift, so the graph is increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$.

(e) $y = f(x + 1)$ is a horizontal shift one unit to the left, so the graph is increasing on $(-\infty, 1)$ and decreasing on $(1, \infty)$.

64. (a) $y = f(-x)$ is a reflection in the y-axis, so the graph is increasing on $(-2, 1)$ and decreasing on $(-\infty, -2)$ and $(1, \infty)$.

(b) $y = -f(x)$ is a reflection in the x-axis, so the graph is increasing on $(-1, 2)$ and decreasing on $(-\infty, -1)$ and $(2, \infty)$.

(c) $y = \frac{1}{2}f(x)$ is a vertical stretch, so the graph is increasing on $(-\infty, -1)$ and $(2, \infty)$, and decreasing on $(-1, 2)$.

(d) $y = -f(x - 1)$ is a horizontal shift and reflection, so the graph is increasing on $(0, 3)$ and decreasing on $(-\infty, 0)$ and $(3, \infty)$.

(e) $y = f(x - 2) + 1$ is a horizontal shift 2 units to the right, and a vertical shift, so the graph is increasing on $(-\infty, 1)$ and $(4, \infty)$, and decreasing on $(1, 4)$.

65. The vertex is approximately at $(2, 1)$ and the graph opens upward. Matches (c).

66. The domain is $[0, -\infty)$ and $(0, -4)$ is approximately on the graph, and $f(x) < 0$. Matches (c).

67. The vertex is approximately $(2, -4)$ and the graph opens upward. Matches (c).

68. The graph of f is $y = x^3$ shifted to the left approximately four units, reflected in the x-axis, and shifted upward approximately two units. Matches (b).

69. Slope L_1: $\dfrac{10 + 2}{2 + 2} = 3$

Slope L_2: $\dfrac{9 - 3}{3 + 1} = \dfrac{3}{2}$

Neither parallel nor perpendicular

70. Slope L_1: $\dfrac{3 - (-7)}{4 - (-1)} = \dfrac{10}{5} = 2$

Slope L_2: $\dfrac{-7 - 5}{-2 - 1} = \dfrac{-12}{-3} = 4$

Neither parallel nor perpendicular

71. Domain: All $x \neq 9$

72. $f(x) = \dfrac{\sqrt{x - 5}}{x - 7}$

Domain: $x \geq 5$ and $x \neq 7$

73. Domain:
$100 - x^2 \geq 0 \Rightarrow x^2 \leq 100 \Rightarrow -10 \leq x \leq 10$

74. $f(x) = \sqrt[3]{16 - x^2}$

Domain: all real numbers

Section 1.5 Combinations of Functions

■ Given two functions, f and g, you should be able to form the following functions (if defined):

1. Sum: $(f + g)(x) = f(x) + g(x)$
2. Difference: $(f - g)(x) = f(x) - g(x)$
3. Product: $(fg)(x) = f(x)g(x)$
4. Quotient: $(f/g)(x) = f(x)/g(x), g(x) \neq 0$
5. Composition of f with g: $(f \circ g)(x) = f(g(x))$
6. Composition of g with f: $(g \circ f)(x) = g(f(x))$

Vocabulary Check

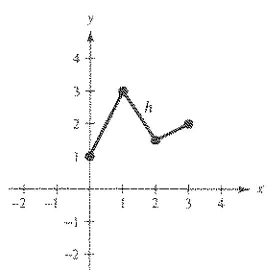
1. addition, subtraction, multiplication, division

2. composition

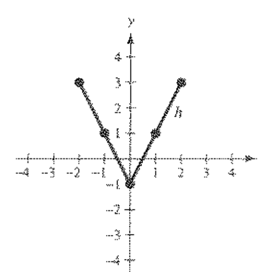
3. $g(x)$

4. inner, outer

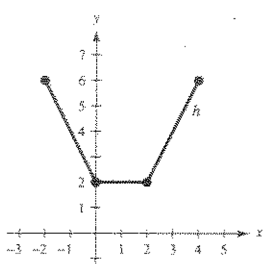
1.



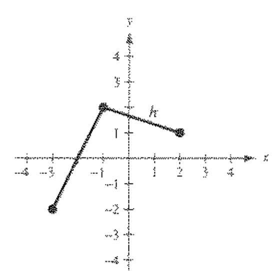
2.



3.



4.



5. $f(x) = x + 3, \quad g(x) = x - 3$

(a) $(f + g)(x) = f(x) + g(x) = (x + 3) + (x - 3) = 2x$

(b) $(f - g)(x) = f(x) - g(x) = (x + 3) - (x - 3) = 6$

(c) $(fg)(x) = f(x)g(x) = (x + 3)(x - 3) = x^2 - 9$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x + 3}{x - 3}, \quad x \neq 3$

Domain: all $x \neq 3$

6. $f(x) = 2x - 5,\ g(x) = 1 - x$

(a) $(f + g)(x) = 2x - 5 + 1 - x = x - 4$

(b) $(f - g)(x) = 2x - 5 - (1 - x)$

$= 2x - 5 - 1 + x$

$= 3x - 6$

(c) $(fg)(x) = (2x - 5)(1 - x)$

$= 2x - 2x^2 - 5 + 5x$

$= -2x^2 + 7x - 5$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{1 - x}$

Domain: $1 - x \neq 0$

$x \neq 1$

7. $f(x) = x^2, g(x) = 1 - x$

(a) $(f + g)(x) = f(x) + g(x) = x^2 + (1 - x) = x^2 - x + 1$

(b) $(f - g)(x) = f(x) - g(x) = x^2 - (1 - x) = x^2 + x - 1$

(c) $(fg)(x) = f(x) \cdot g(x) = x^2(1 - x) = x^2 - x^3$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{f(x)}{g(x)} = \dfrac{x^2}{1 - x}, x \neq 1$

Domain: all $x \neq 1$.

8. $f(x) = 2x - 5,\ g(x) = 4$

(a) $(f + g)(x) = 2x - 5 + 4 = 2x - 1$

(b) $(f - g)(x) = 2x - 5 - 4 = 2x - 9$

(c) $(fg)(x) = (2x - 5)(4) = 8x - 20$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{2x - 5}{4} = \dfrac{1}{2}x - \dfrac{5}{4}$

Domain: $-\infty < x < \infty$

9. $f(x) = x^2 + 5, g(x) = \sqrt{1 - x}$

(a) $(f + g)(x) = x^2 + 5 + \sqrt{1 - x}$

(b) $(f - g)(x) = x^2 + 5 - \sqrt{1 - x}$

(c) $(fg)(x) = (x^2 + 5)\sqrt{1 - x}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x^2 + 5}{\sqrt{1 - x}}$

Domain: $x < 1$

10. $f(x) = \sqrt{x^2 - 4},\ g(x) = \dfrac{x^2}{x^2 + 1}$

(a) $(f + g)(x) = \sqrt{x^2 - 4} + \dfrac{x^2}{x^2 + 1}$

(b) $(f - g)(x) = \sqrt{x^2 - 4} - \dfrac{x^2}{x^2 + 1}$

(c) $(fg)(x) = \left(\sqrt{x^2 - 4}\right)\left(\dfrac{x^2}{x^2 + 1}\right) = \dfrac{x^2\sqrt{x^2 - 4}}{x^2 + 1}$

Domain: $x^2 - 4 \geq 0$

$x^2 \geq 4 \implies x \geq 2 \text{ or } x \leq -2$

(d) $\left(\dfrac{f}{g}\right)(x) = \sqrt{x^2 - 4} \div \dfrac{x^2}{x^2 + 1}$

$= \dfrac{(x^2 + 1)\sqrt{x^2 - 4}}{x^2}$

Domain: $x^2 - 4 \geq 0$ and $x \neq 0$

$x \geq 2$ or $x \leq -2$

11. $f(x) = \dfrac{1}{x}, g(x) = \dfrac{1}{x^2}$

(a) $(f + g)(x) = \dfrac{1}{x} + \dfrac{1}{x^2} = \dfrac{x + 1}{x^2}$

(b) $(f - g)(x) = \dfrac{1}{x} - \dfrac{1}{x^2} = \dfrac{x - 1}{x^2}$

(c) $(fg)(x) = \dfrac{1}{x} \cdot \dfrac{1}{x^2} = \dfrac{1}{x^3}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{1/x}{1/x^2} = x, x \neq 0$

Domain: $x \neq 0$

12. $f(x) = \dfrac{x}{x+1},\ g(x) = x^3$

(a) $(f+g)(x) = \dfrac{x}{x+1} + x^3 = \dfrac{x + x^4 + x^3}{x+1}$

(b) $(f-g)(x) = \dfrac{x}{x+1} - x^3 = \dfrac{x - x^4 - x^3}{x+1}$

(c) $(fg)(x) = \dfrac{x}{x+1} \cdot x^3 = \dfrac{x^4}{x+1}$

(d) $\left(\dfrac{f}{g}\right)(x) = \dfrac{x}{x+1} \div x^3$

$= \dfrac{x}{x+1} \cdot \dfrac{1}{x^3} = \dfrac{1}{x^2(x+1)}$

Domain: $x \neq 0, x \neq -1$

13. $(f+g)(3) = f(3) + g(3)$

$= (3^2 - 1) + (3 - 2)$

$= 8 + 1 = 9$

14. $(f-g)(-2) = f(-2) - g(-2)$

$= ((-2)^2 - 1) - (-2 - 2)$

$= 3 - (-4) = 7$

15. $(f-g)(0) = f(0) - g(0)$

$= (0 - 1) - (0 - 2)$

$= 1$

16. $(f+g)(1) = f(1) + g(1)$

$= (1 - 1) + (1 - 2)$

$= -1$

17. $(fg)(4) = f(4)g(4)$

$= (4^2 - 1)(4 - 2)$

$= 15(2)$

$= 30$

18. $(fg)(-6) = f(-6)g(-6)$

$= ((-6)^2 - 1)(-6 - 2)$

$= 35(-8)$

$= -280$

19. $\left(\dfrac{f}{g}\right)(-5) = \dfrac{f(-5)}{g(-5)}$

$= \dfrac{(-5)^2 - 1}{-5 - 2}$

$= \dfrac{24}{-7}$

$= -\dfrac{24}{7}$

20. $\left(\dfrac{f}{g}\right)(0) = \dfrac{f(0)}{g(0)}$

$= \dfrac{0 - 1}{0 - 2}$

$= \dfrac{1}{2}$

21. $(f-g)(2t) = f(2t) - g(2t)$

$= ((2t)^2 - 1) - (2t - 2)$

$= 4t^2 - 2t + 1$

22. $(f+g)(t-4) = f(t-4) + g(t-4)$

$= ((t-4)^2 - 1) + (t - 4 - 2)$

$= t^2 - 8t + 15 + t - 6$

$= t^2 - 7t + 9$

23. $(fg)(-5t) = f(-5t)g(-5t)$

$= ((-5t)^2 - 1)(-5t - 2)$

$= (25t^2 - 1)(-5t - 2)$

$= -125t^3 - 50t^2 + 5t + 2$

24. $(fg)(3t^2) = f(3t^2)g(3t^2)$

$= ((3t^2)^2 - 1)(3t^2 - 2)$

$= (9t^4 - 1)(3t^2 - 2)$

$= 27t^6 - 18t^4 - 3t^2 + 2$

25. $\left(\dfrac{f}{g}\right)(-t) = \dfrac{f(-t)}{g(-t)}$

$= \dfrac{(-t)^2 - 1}{-t - 2}$

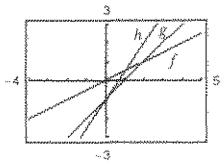
$= \dfrac{t^2 - 1}{-t - 2} = \dfrac{1 - t^2}{t + 2}, \quad t \neq -2$

26. $\left(\dfrac{f}{g}\right)(t+2) = \dfrac{f(t+2)}{g(t+2)}$

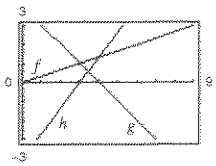
$= \dfrac{(t+2)^2 - 1}{(t+2) - 2}$

$= \dfrac{t^2 + 4t + 3}{t}, \quad t \neq 0$

27.



28.



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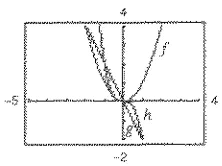
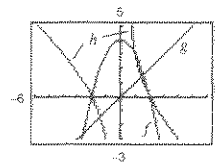
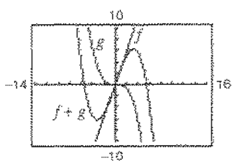


30.



31. $f(x) = 3x,\ g(x) = -\dfrac{x^3}{10},\ (f+g)(x) = 3x - \dfrac{x^3}{10}$

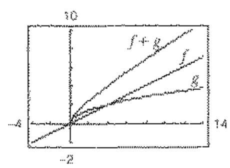


For $0 \leq x \leq 2$, $f(x)$ contributes more to the magnitude.

For $x > 6$, $g(x)$ contributes more to the magnitude.

32. $f(x) = \dfrac{x}{2},\ g(x) = \sqrt{x}$

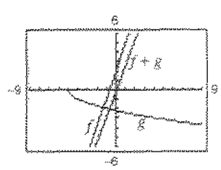
$(f+g)(x) = \dfrac{x}{2} + \sqrt{x}$



$g(x)$ contributes more to the magnitude of the sum for $0 \leq x \leq 2$. $f(x)$ contributes more to the magnitude of the sum for $x > 6$.

33. $f(x) = 3x + 2,\ g(x) = -\sqrt{x+5},$

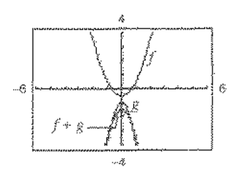
$(f+g)(x) = 3x + 2 - \sqrt{x+5}$



$f(x) = 3x + 2$ contributes more to the magnitude in both intervals.

34. $f(x) = x^2 - \frac{1}{2},\ g(x) = -3x^2 - 1,$

$(f+g)(x) = \left(x^2 - \frac{1}{2}\right) + (-3x^2 - 1) = -2x^2 - \frac{3}{2}$



g contributes more on both intervals.

35. $f(x) = x^2, g(x) = x - 1$

(a) $(f \circ g)(x) = f(g(x)) = f(x - 1) = (x - 1)^2$

(b) $(g \circ f)(x) = g(f(x)) = g(x^2) = x^2 - 1$

(c) $(f \circ g)(0) = (0 - 1)^2 = 1$

36. $f(x) = \sqrt[3]{x - 1},\ g(x) = x^3 + 1$

(a) $(f \circ g)(x) = f(g(x))$

$= f(x^3 + 1)$

$= \sqrt[3]{(x^3 + 1) - 1}$

$= \sqrt[3]{x^3} = x$

(b) $(g \circ f)(x) = g(f(x))$

$= g\left(\sqrt[3]{x - 1}\right)$

$= \left(\sqrt[3]{x - 1}\right)^3 + 1$

$= (x - 1) + 1 = x$

(c) $(f \circ g)(0) = 0$

37. $f(x) = 3x + 5, g(x) = 5 - x$

(a) $(f \circ g)(x) = f(g(x)) = f(5 - x) = 3(5 - x) + 5 = 20 - 3x$

(b) $(g \circ f)(x) = g(f(x)) = g(3x + 5) = 5 - (3x + 5) = -3x$

(c) $(f \circ g)(0) = 20$

38. $f(x) = x^3,\ g(x) = \dfrac{1}{x}$

(a) $(f \circ g)(x) = f(g(x)) = f\left(\dfrac{1}{x}\right) = \left(\dfrac{1}{x}\right)^3 = \dfrac{1}{x^3}$

(b) $(g \circ f)(x) = g(f(x)) = g(x^3) = \dfrac{1}{x^3}$

(c) $(f \circ g)(0)$ is not defined.

39. (a) The domain of $f(x) = \sqrt{x + 4}$ is $x + 4 \geq 0$ or $x \geq -4$.

(b) The domain of $g(x) = x^2$ is all real numbers.

(c) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$.

The domain of $(f \circ g)$ is all real numbers.

40. (a) Domain of f: $x + 3 \geq 0 \Rightarrow x \geq -3$

(b) Domain of g: all real numbers

(c) Domain of $(f \circ g)(x) = f\left(\dfrac{x}{2}\right) = \sqrt{\dfrac{x}{2} + 3}$:

$\dfrac{x}{2} + 3 \geq 0 \Rightarrow x \geq -6$

41. (a) The domain of $f(x) = x^2 + 1$ is all real numbers.

(b) The domain of $g(x) = \sqrt{x}$ is all $x \geq 0$.

(c) $(f \circ g)(x) = f(g(x)) = f\left(\sqrt{x}\right)$

$= \left(\sqrt{x}\right)^2 + 1 = x + 1,\quad x \geq 0$

The domain of $f \circ g$ is $x \geq 0$.

42. $f(x) = x^{1/4},\quad g(x) = x^4$

(a) Domain of f: $x \geq 0$

(b) Domain of g: all x

(c) $(f \circ g)(x) = f(g(x)) = f(x^4) = (x^4)^{1/4} = x$

Domain: all x

43. (a) The domain of $f(x) = \frac{1}{x}$ is all $x \neq 0$.

(b) The domain of $g(x) = x + 3$ is all real numbers.

(c) The domain of $(f \circ g)(x) = f(x + 3) = \frac{1}{x + 3}$ is all $x \neq -3$.

44. (a) Domain of f: all $x \neq 0$

(b) Domain of g: all $x \neq 0$

(c) Domain of $(f \circ g)(x) = f\left(\frac{1}{2x}\right) = 2x, \quad x \neq 0,$ is all $x \neq 0$.

45. (a) The domain of $f(x) = |x - 4|$ is all real numbers.

(b) The domain of $g(x) = 3 - x$ is all real numbers.

(c) $(f \circ g)(x) = f(g(x)) = f(3 - x) = |(3 - x) - 4| = |-x - 1| = |x + 1|$

Domain: all real numbers

46. $f(x) = \frac{2}{|x|}, \quad g(x) = x - 1$

(a) Domain of f: all $x \neq 0$

(b) Domain of g: all x

(c) $(f \circ g)(x) = f(g(x)) = f(x - 1) = \frac{2}{|x - 1|}$

Domain: all $x \neq 1$

47. (a) The domain of $f(x) = x + 2$ is all real numbers.

(b) The domain of $g(x) = \frac{1}{x^2 - 4}$ is all $x \neq \pm 2$

(c) $(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x^2 - 4}\right) = \frac{1}{x^2 - 4} + 2$

Domain: $x \neq \pm 2$

48. (a) Domain of f: all $x \neq \pm 1$

(b) Domain of g: all real numbers

(c) Domain of $(f \circ g)(x) = f(x + 1) = \frac{3}{(x + 1)^2 - 1}$

$$= \frac{3}{x^2 + 2x} = \frac{3}{x(x + 2)}$$

is all real numbers $\neq 0, -2$.

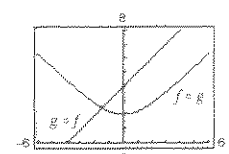
49. (a) $(f \circ g)(x) = f(g(x)) = f(x^2) = \sqrt{x^2 + 4}$

Domain: all x

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x + 4}\right) = \left(\sqrt{x + 4}\right)^2$

$= x + 4, \ x \geq -4$

(b)



They are not equal.

50. (a) $(f \circ g)(x) = f(g(x)) = f(x^3 - 1)$

$= \sqrt[3]{(x^3 - 1) + 1} = \sqrt[3]{x^3} = x$

Domain: all x

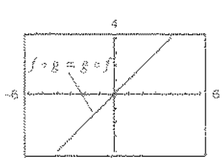
$(g \circ f)(x) = g(f(x)) = g\left(\sqrt[3]{x + 1}\right)$

$= \left[\sqrt[3]{x + 1}\right]^3 - 1$

$= (x + 1) - 1 = x$

They are equal. $(f \circ g)(x) = (g \circ f)(x) = x$

(b)



51. (a) $(f \circ g)(x) = f(g(x)) = f(3x + 9)$

$= \frac{1}{3}(3x + 9) - 3 = x$

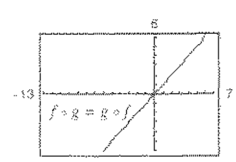
Domain: all x

$(g \circ f)(x) = g(f(x)) = g\left(\frac{1}{3}x - 3\right)$

$= 3\left(\frac{1}{3}x - 3\right) + 9 = x$

The domain of $f \circ g$ is all real numbers.

(b)

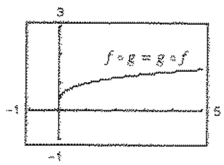


They are equal.

52. (a) $(f \circ g)(x) = (g \circ f)(x) = \sqrt{\sqrt{x}} = x^{1/4}$

Domain: $x \geq 0$

(b)



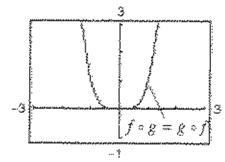
They are equal.

53. (a) $(f \circ g)(x) = f(g(x)) = f(x^6) = (x^6)^{2/3} = x^4$

Domain: all x

$(g \circ f)(x) = g(f(x)) = g(x^{2/3}) = (x^{2/3})^6 = x^4$

(b)



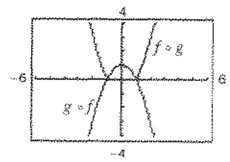
They are equal.

54. (a) $(f \circ g)(x) = f(g(x)) = f(-x^2 + 1) = |-x^2 + 1|$

Domain: all x

$(g \circ f)(x) = g(f(x)) = g(|x|) = -|x|^2 + 1$

(b)



$f \circ g \neq g \circ f$

55. (a) $(f \circ g)(x) = f(g(x)) = f(4 - x) = 5(4 - x) + 4 = 24 - 5x$

$(g \circ f)(x) = g(f(x)) = g(5x + 4) = 4 - (5x + 4) = -5x$

(b) No, $(f \circ g)(x) \neq (g \circ f)(x)$ because $24 - 5x \neq -5x$.

(c)

x	$f(g(x))$	$g(f(x))$
0	24	0
1	19	−5
2	14	−10
3	9	−15

56. (a) $(f \circ g)(x) = f(4x + 1) = \frac{1}{4}[(4x + 1) - 1] = \frac{1}{4}[4x] = x$

$(g \circ f)(x) = g\left(\frac{1}{4}(x - 1)\right) = 4\left[\frac{1}{4}(x - 1)\right] + 1 = (x - 1) + 1 = x$

(b) They are equal because $x = x$.

(c)

x	$f(g(x))$	$g(f(x))$
−1	−1	−1
0	0	0
1	1	1
2	2	2
3	3	3

57. (a) $(f \circ g)(x) = f(g(x)) = f(x^2 - 5) = \sqrt{(x^2 - 5) + 6} = \sqrt{x^2 + 1}$

$(g \circ f)(x) = g(f(x)) = g\left(\sqrt{x + 6}\right) = \left(\sqrt{x + 6}\right)^2 - 5$

$= (x + 6) - 5 = x + 1, x \geq -6$

(b) No, $(f \circ g)(x) \neq (g \circ f)(x)$ because $\sqrt{x^2 + 1} \neq x + 1$.

(c)

x	$f(g(x))$	$g(f(x))$
0	1	1
−2	$\sqrt{5}$	−1
3	$\sqrt{10}$	4

58. (a) $(f \circ g)(x) = f\left(\sqrt[3]{x + 10}\right) = \left[\sqrt[3]{x + 10}\right]^3 - 4$

$= (x + 10) - 4 = x + 6$

$(g \circ f)(x) = g(x^3 - 4) = \sqrt[3]{(x^3 - 4) + 10} = \sqrt[3]{x^3 + 6}$

(b) They are not equal because $x + 6 \neq \sqrt[3]{x^3 + 6}$.

(c)

x	$f(g(x))$	$g(f(x))$
−2	4	$\sqrt[3]{-2}$
0	6	$\sqrt[3]{6}$
1	7	$\sqrt[3]{7}$
2	8	$\sqrt[3]{14}$
3	9	$\sqrt[3]{33}$

59. (a) $(f \circ g)(x) = f(g(x)) = f(2x - 1) = |(2x - 1) + 3|$

$= |2x + 2| = 2|x + 1|$

$(g \circ f)(x) = g(f(x)) = g(|x + 3|) = 2|x + 3| - 1$

(b) No, $(f \circ g)(x) \neq (g \circ f)(x)$ because $2|x + 1| \neq 2|x + 3| - 1$.

(c)

x	$f(g(x))$	$g(f(x))$
-1	0	3
0	2	5
1	4	7

60. (a) $(f \circ g)(x) = f(g(x)) = f(-x) = \dfrac{6}{3(-x) - 5} = \dfrac{6}{-3x - 5}$

$(g \circ f)(x) = g\left(\dfrac{6}{3x - 5}\right) = -\left(\dfrac{6}{3x - 5}\right) = \dfrac{-6}{3x - 5}$

(b) They are not equal because $\dfrac{6}{-3x - 5} \neq \dfrac{-6}{3x - 5}$.

(c)

x	$f(g(x))$	$g(f(x))$
0	$-\frac{6}{5}$	$\frac{6}{5}$
1	$-\frac{3}{4}$	3
2	$-\frac{6}{11}$	-6
3	$-\frac{3}{7}$	$-\frac{3}{2}$

61. (a) $(f + g)(3) = f(3) + g(3) = 2 + 1 = 3$

(b) $\left(\dfrac{f}{g}\right)(2) = \dfrac{f(2)}{g(2)} = \dfrac{0}{2} = 0$

62. (a) $(f - g)(1) = f(1) - g(1) = 2 - 3 = -1$

(b) $(fg)(4) = f(4) \cdot g(4) = 4 \cdot 0 = 0$

63. (a) $(f \circ g)(2) = f(g(2)) = f(2) = 0$

(b) $(g \circ f)(2) = g(f(2)) = g(0) = 4$

64. (a) $(f \circ g)(1) = f(g(1)) = f(3) = 2$

(b) $(g \circ f)(3) = g(f(3)) = g(2) = 2$

65. Let $f(x) = x^2$ and $g(x) = 2x + 1$, then $(f \circ g)(x) = h(x)$. This is not a unique solution. For example, if $f(x) = (x + 1)^2$ and $g(x) = 2x$, then $(f \circ g)(x) = h(x)$ as well.

66. $h(x) = (1 - x)^3$

One possibility: Let $g(x) = 1 - x$ and $f(x) = x^3$.

$(f \circ g)(x) = f(1 - x) = (1 - x)^3 = h(x)$

67. Let $f(x) = \sqrt[3]{x}$ and $g(x) = x^2 - 4$, then $(f \circ g)(x) = h(x)$. This answer is not unique. Other possibilities may be:

$f(x) = \sqrt[3]{x - 4}$ and $g(x) = x^2$ or

$f(x) = \sqrt[3]{-x}$ and $g(x) = 4 - x^2$ or

$f(x) = \sqrt[9]{x}$ and $g(x) = (x^2 - 4)^3$

68. $h(x) = \sqrt{9 - x}$

One possibility: Let $g(x) = 9 - x$ and $f(x) = \sqrt{x}$.

$(f \circ g)(x) = f(9 - x) = \sqrt{9 - x} = h(x)$

69. Let $f(x) = 1/x$ and $g(x) = x + 2$, then $(f \circ g)(x) = h(x)$. Again, this is not a unique solution. Other possibilities may be:

$f(x) = \dfrac{1}{x + 2}$ and $g(x) = x$

or $f(x) = \dfrac{1}{x + 1}$ and $g(x) = x + 1$

70. $h(x) = \dfrac{4}{(5x+2)^2}$

One possibility:

Let $g(x) = 5x + 2$ and $f(x) = \dfrac{4}{x^2}$.

$(f \circ g)(x) = f(5x+2) = \dfrac{4}{(5x+2)^2}$

71. Let $f(x) = x^2 + 2x$ and $g(x) = x + 4$. Then $(f \circ g)(x) = h(x)$. (Answer is not unique.)

72. $h(x) = (x+3)^{3/2} + 4(x+3)^{1/2}$

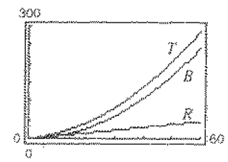
One possibility:

Let $g(x) = x + 3$ and $f(x) = x^{3/2} + 4x^{1/2}$.

$$(f \circ g)(x) = f(g(x)) = f(x+3)$$
$$= (x+3)^{3/2} + 4(x+3)^{1/2} = h(x)$$

73. (a) $T(x) = R(x) + B(x) = \frac{3}{4}x + \frac{1}{15}x^2$

(b)

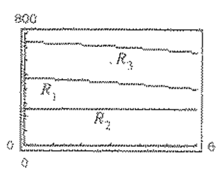


(c) $B(x)$ contributes more to $T(x)$ at higher speeds.

74. (a) $R_3 = R_1 + R_2$

$$= (480 - 8t - 0.8t^2) + (254 + 0.78t)$$
$$= 734 - 7.22t - 0.8t^2, \quad t = 0, 1, 2, 3, 4, 5, 6$$

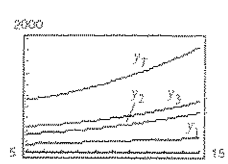
(b)



75. $t = 5$ corresponds to 1995.

Year	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
y_1	140	151.4	162.8	174.2	185.6	197	208.4	219.8	231.2	242.6	254
y_2	325.8	342.8	364.4	390.6	421.5	457	497.1	541.8	591.2	645.2	703.8
y_3	458.8	475.3	497.9	526.5	561.2	602	648.8	701.7	760.7	825.7	896.8

76.



y_T represents the total out-of-pocket payments, insurance premiums and other types of premiums in billions of dollars.

77. $(A \circ r)(t)$ gives the area of the circle as a function of time.

$$(A \circ r)(t) = A(r(t))$$
$$= A(0.6t)$$
$$= \pi(0.6t)^2 = 0.36\pi t^2$$

78. (a) $r(x) = \dfrac{x}{2}$

(b) $A(r) = \pi r^2$

(c) $(A \circ r)(x) = A(r(x))$

$$= A\left(\frac{x}{2}\right) = \pi\left(\frac{x}{2}\right)^2 = \frac{1}{4}\pi x^2$$

$A \circ r$ represents the area of the circular base of the tank with edge x.

79. $C(x) = 60x + 750$

$x(t) = 50t$

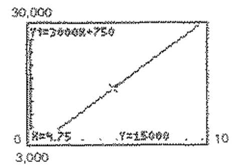
(a) $C(x(t)) = C(50t)$

$= 60(50t) + 750$

$= 3000t + 750$

$C(x(t))$ represents the cost after t hours.

(b) $x(4) = 50(4) = 200$ units

(c) 

$t = 4.75$, or 4 hours 45 minutes

80. $x = 150$ miles $-$ (450 mph)(t hours)

$y = 200$ miles $-$ (450 mph)(t hours)

$$s = \sqrt{x^2 + y^2} = \sqrt{(150 - 450t)^2 + (200 - 450t)^2} = 50\sqrt{162t^2 - 126t + 25}$$

81. (a) $(N \circ T)(t) = N(T(t))$

$= N(2t + 1)$

$= 10(2t + 1)^2 - 20(2t + 1) + 600$

$= 40t^2 + 590$

$N \circ T$ represents the number of bacteria as a function of time.

(b) $(N \circ T)(6) = 10(13^2) - 20(13) + 600 = 2030$

At time $t = 6$, there are 2030 bacteria.

(c) $N = 800$ when $t \approx 2.3$ hours.

82. (a) Area $= \pi r^2$, $r(t) = 5.25\sqrt{t}$. Hence

$(A \circ r)(t) = \pi\left[5.25\sqrt{t}\right]^2 = 27.5625\pi t, \quad t \geq 0$

(b) $(A \circ r)(36) = 27.5625\pi(36) = 992.25\pi$

≈ 3117 square meters

(c) $A = 6250 = 27.5625\pi t \implies t \approx 72.2$ hours

83. $g(f(x)) = g(x - 500{,}000) = 0.03(x - 500{,}000)$ represents 3 percent of the amount over \$500,000.

84. (a) $R = p - 1200$

(b) $S = 0.92p$

(c) $(R \circ S)(p) = 0.92p - 1200$

$(S \circ R)(p) = 0.92(p - 1200)$

(d) $(R \circ S)(18{,}400) = 15{,}728$

$(S \circ R)(18{,}400) = 15{,}824$

The discount first yields a lower cost.

85. False. $(f \circ g)(x) = f(6x) = 6x + 1$, but $(g \circ f)(x) = g(x + 1) = 6(x + 1)$.

86. True. $(f \circ g)(x) = f(g(x))$ is only defined if $g(x)$ is in the domain of f.

87. Let A, B, and C be the three siblings, in decreasing age. Then $A = 2B$ and $B = \frac{1}{2}C + 6$.

(a) $A = 2B = 2\left(\frac{1}{2}C + 6\right) = C + 12$

(b) If $A = 16$, then $B = 8$ and $C = 4$.

88. From Exercise 87, $A = 2B$ and $B = \frac{1}{2}C + 6$.

(a) $2(B - 6) = C$ and $B = \frac{1}{2}A$. Hence,

$C = 2\left(\frac{1}{2}A - 6\right) = A - 12$.

(b) If $C = 2$, then $B = 7$ and $A = 14$.

89. Let $f(x)$ and $g(x)$ be odd functions, and define $h(x) = f(x)g(x)$. Then,

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= [-f(x)][-g(x)] \quad \text{since } f \text{ and } g \text{ are both odd} \\ &= f(x)g(x) = h(x). \end{aligned}$$

Thus, h is even.

Let $f(x)$ and $g(x)$ be even functions, and define $h(x) = f(x)g(x)$. Then,

$$\begin{aligned} h(-x) &= f(-x)g(-x) \\ &= f(x)g(x) \quad \text{since } f \text{ and } g \text{ are both even} \\ &= h(x). \end{aligned}$$

Thus, h is even.

90. The product of an odd function and an even function is odd. Let f be odd and g even. Then

$(fg)(-x) = f(-x)g(-x) = -f(x)g(x) = -(fg)(x)$

Thus, fg is odd.

91. $g(-x) = \frac{1}{2}[f(-x) + f(-(-x))] = \frac{1}{2}[f(-x) + f(x)] = g(x)$,

which shows that g is even.

$$\begin{aligned} h(-x) &= \tfrac{1}{2}[f(-x) - f(-(-x))] = \tfrac{1}{2}[f(-x) - f(x)] \\ &= -\tfrac{1}{2}[f(x) - f(-x)] = -h(x), \end{aligned}$$

which shows that h is odd.

92. (a) $$\begin{aligned} f(x) &= \frac{1}{2}[f(x) + f(-x)] + \frac{1}{2}[f(x) - f(-x)] \\ &= \qquad g(x) \qquad + \qquad h(x) \end{aligned}$$

where g is even and h is odd.

(b) $$\begin{aligned} f(x) &= \frac{1}{2}[(x^2 - 2x + 1) + (x^2 + 2x + 1)] + \frac{1}{2}[(x^2 - 2x + 1) - (x^2 + 2x + 1)] \\ &= \frac{1}{2}[2x^2 + 2] + \frac{1}{2}[-4x] = [x^2 + 1] + [-2x] \end{aligned}$$

$$\begin{aligned} g(x) &= \frac{1}{2}\left[\frac{1}{x+1} + \frac{1}{-x+1}\right] + \frac{1}{2}\left[\frac{1}{x+1} - \frac{1}{-x+1}\right] \\ &= \frac{-1}{(x+1)(x-1)} + \frac{x}{(x+1)(x-1)} \end{aligned}$$

93. $(0, -5), (1, -5), (2, -7)$ (other answers possible)

94. Three points on the graph of $y = \frac{1}{5}x^3 - 4x^2 + 1$ are $(0, 1), (1, -2.8)$ and $(-1, -3.2)$.

95. $(\sqrt{24}, 0), (-\sqrt{24}, 0), (0, \sqrt{24})$
(other answers possible)

96. Three points on the graph of $y = \dfrac{x}{x^2 - 5}$ are $(0, 0), \left(1, -\dfrac{1}{4}\right)$ and $\left(-1, \dfrac{1}{4}\right)$.

97.
$$y - (-2) = \frac{8 - (-2)}{-3 - (-4)}(x - (-4))$$
$$y + 2 = 10(x + 4)$$
$$y - 10x - 38 = 0$$

98.
$$y - 5 = \frac{2 - 5}{-8 - 1}(x - 1)$$
$$y - 5 = \frac{1}{3}(x - 1)$$
$$3y - x - 14 = 0$$

99.
$$y - (-1) = \frac{4 - (-1)}{-(1/3) - (3/2)}\left(x - \frac{3}{2}\right)$$
$$y + 1 = \frac{5}{-11/6}\left(x - \frac{3}{2}\right) = -\frac{30}{11}\left(x - \frac{3}{2}\right)$$
$$11y + 11 = -30x + 45$$
$$30x + 11y - 34 = 0$$

100.
$$y - 1.1 = \frac{3.1 - 1.1}{-4 - 0}(x - 0)$$
$$y - 1.1 = -\frac{1}{2}x$$
$$2y + x - 2.2 = 0$$

Section 1.6 Inverse Functions

- Two functions f and g are inverses of each other if $f(g(x)) = x$ for every x in the domain of g and $g(f(x)) = x$ for every x in the domain of f.
- Be able to find the inverse of a function, if it exists.
  1. Replace $f(x)$ with y.
  2. Interchange x and y.
  3. Solve for y. If this equation represents y as a function of x, then you have found $f^{-1}(x)$. If this equation does not represent y as a function of x, then f does not have an inverse function.
- A function f has an inverse function if and only if no **horizontal** line crosses the graph of f at more than one point.
- A function f has an inverse function if and only if f is one-to-one.

Vocabulary Check

1. inverse, f^{-1} **2.** range, domain **3.** $y = x$

4. one-to-one **5.** Horizontal

1.
$$f(x) = 6x$$
$$f^{-1}(x) = \tfrac{1}{6}x$$
$$f(f^{-1}(x)) = f\left(\tfrac{1}{6}x\right) = 6\left(\tfrac{1}{6}x\right) = x$$
$$f^{-1}(f(x)) = f^{-1}(6x) = \tfrac{1}{6}(6x) = x$$

2.
$$f(x) = \tfrac{1}{3}x$$
$$f^{-1}(x) = 3x$$
$$f(f^{-1}(x)) = f(3x) = \tfrac{1}{3}(3x) = x$$
$$f^{-1}(f(x)) = f^{-1}\left(\tfrac{1}{3}x\right) = 3\left(\tfrac{1}{3}x\right) = x$$

3. $f(x) = x + 7$

$f^{-1}(x) = x - 7$

$f(f^{-1}(x)) = f(x - 7) = (x - 7) + 7 = x$

$f^{-1}(f(x)) = f^{-1}(x + 7) = (x + 7) - 7 = x$

4. $f(x) = x - 3$

$f^{-1}(x) = x + 3$

$f(f^{-1}(x)) = f(x + 3) = (x + 3) - 3 = x$

$f^{-1}(f(x)) = f^{-1}(x - 3) = (x - 3) + 3 = x$

5. $f^{-1}(x) = \dfrac{x - 1}{2}$

$f(f^{-1}(x)) = f\left(\dfrac{x-1}{2}\right) = 2\left(\dfrac{x-1}{2}\right) + 1 = (x - 1) + 1 = x$

$f^{-1}(f(x)) = f^{-1}(2x + 1) = \dfrac{(2x + 1) - 1}{2} = \dfrac{2x}{2} = x$

6. $f(x) = \dfrac{x - 1}{4}$

$f^{-1}(x) = 4x + 1$

$f(f^{-1}(x)) = f(4x + 1) = \dfrac{(4x + 1) - 1}{4} = \dfrac{4x}{4} = x$

7. $f^{-1}(x) = x^3$

$f(f^{-1}(x)) = f(x^3) = \sqrt[3]{x^3} = x$

$f^{-1}(f(x)) = f^{-1}\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

8. $f(x) = x^5$

$f^{-1}(x) = \sqrt[5]{x}$

$f(f^{-1}(x)) = f\left(\sqrt[5]{x}\right) = \left(\sqrt[5]{x}\right)^5 = x$

$f^{-1}(f(x)) = f^{-1}(x^5) = \sqrt[5]{x^5} = x$

9. (a) $f(g(x)) = f\left(-\dfrac{2x + 6}{7}\right) = -\dfrac{7}{2}\left(-\dfrac{2x + 6}{7}\right) - 3 = \dfrac{2x + 6}{2} - 3 = (x + 3) - 3 = x$

$g(f(x)) = g\left(-\dfrac{7}{2}x - 3\right) = -\dfrac{2\left(-\frac{7}{2}x - 3\right) + 6}{7} = -\dfrac{-7x - 6 + 6}{7} = \dfrac{7x}{7} = x$

(b)

x	2	0	-2	-4	-6
$f(x)$	-10	-3	4	11	18

x	-10	-3	4	11	18
$g(x)$	2	0	-2	-4	-6

Note that the entries in the tables are the same except that the rows are interchanged.

10. (a) $f(g(x)) = f(4x + 9) = \dfrac{(4x + 9) - 9}{4} = \dfrac{4x}{4} = x$

$g(f(x)) = g\left(\dfrac{x - 9}{4}\right) = 4\left(\dfrac{x - 9}{4}\right) + 9$

$= (x - 9) + 9 = x$

(b)

x	1	5	9	13	17
$f(x)$	-2	-1	0	1	2

x	-2	-1	0	1	2
$g(x)$	1	5	9	13	17

The entries are the same except that the rows are interchanged.

11. (a) $f(g(x)) = f\left(\sqrt[3]{x-5}\right) = \left[\sqrt[3]{x-5}\right]^3 + 5 = (x-5) + 5 = x$

$g(f(x)) = g(x^3+5) = \sqrt[3]{(x^3+5)-5} = \sqrt[3]{x^3} = x$

(b)

x	-3	-2	-1	0	1
$f(x)$	-22	-3	4	5	6

x	-22	-3	4	5	6
$g(x)$	-3	-2	-1	0	1

Note that the entries in the tables are the same except that the rows are interchanged.

12. (a) $f(g(x)) = f\left(\sqrt[3]{2x}\right) = \dfrac{\left(\sqrt[3]{2x}\right)^3}{2} = \dfrac{2x}{2} = x$

$g(f(x)) = g\left(\dfrac{x^3}{2}\right) = \sqrt[3]{2\left(\dfrac{x^3}{2}\right)} = \sqrt[3]{x^3} = x$

(b)

x	-2	-1	0	1	2
$f(x)$	-4	$-\frac{1}{2}$	0	$\frac{1}{2}$	4

x	-4	$-\frac{1}{2}$	0	$\frac{1}{2}$	4
$g(x)$	-2	-1	0	1	2

The entries are the same except that the rows are interchanged.

13. (a) $f(g(x)) = f(8+x^2) = -\sqrt{(8+x^2)-8} = -\sqrt{x^2} = -(-x) = x \quad x \le 0$

[Since $x \le 0$, $\sqrt{x^2} = -x$]

$g(f(x)) = g\left(-\sqrt{x-8}\right) = 8 + \left[-\sqrt{x-8}\right]^2 = 8 + (x-8) = x$

(b)

x	8	9	12	17	24
$f(x)$	0	-1	-2	-3	-4

x	0	-1	-2	-3	-4
$g(x)$	8	9	12	17	24

Note that the entries in the tables are the same except that the rows are interchanged.

14. (a) $f(g(x)) = f\left(\dfrac{x^3+10}{3}\right) = \sqrt[3]{3\left(\dfrac{x^3+10}{3}\right) - 10} = \sqrt[3]{(x^3+10)-10} = \sqrt[3]{x^3} = x$

$g(f(x)) = g\left(\sqrt[3]{3x-10}\right) = \dfrac{\left[\sqrt[3]{3x-10}\right]^3 + 10}{3} = \dfrac{(3x-10)+10}{3} = \dfrac{3x}{3} = x$

(b)

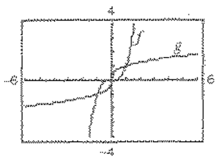
x	$\frac{2}{3}$	3	$\frac{10}{3}$	$\frac{11}{3}$	6
$f(x)$	-2	-1	0	1	2

x	-2	-1	0	1	2
$g(x)$	$\frac{2}{3}$	3	$\frac{10}{3}$	$\frac{11}{3}$	6

The entries in the table are the same except that the rows are interchanged.

15. $f(g(x)) = f\left(\sqrt[3]{x}\right) = \left(\sqrt[3]{x}\right)^3 = x$

$g(f(x)) = g(x^3) = \sqrt[3]{x^3} = x$

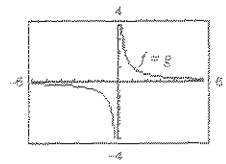


Reflections in the line $y = x$

16. $f(x) = \dfrac{1}{x},\ g(x) = \dfrac{1}{x}$

$f(g(x)) = f\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$

$g(f(x)) = g\left(\dfrac{1}{x}\right) = \dfrac{1}{1/x} = 1 \div \dfrac{1}{x} = 1 \cdot \dfrac{x}{1} = x$



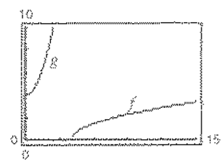
Reflections in the line $y = x$

17. $f(g(x)) = f(x^2 + 4),\ x \ge 0$

$= \sqrt{(x^2 + 4) - 4} = x$

$g(f(x)) = g\left(\sqrt{x - 4}\right)$

$= \left(\sqrt{x - 4}\right)^2 + 4 = x$



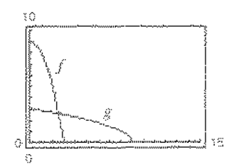
Reflections in the line $y = x$

18. $f(x) = 9 - x^2, x \ge 0$

$g(x) = \sqrt{9 - x}, x \le 9$

$f(g(x)) = f\left(\sqrt{9 - x}\right) = 9 - \left(\sqrt{9 - x}\right)^2 = 9 - (9 - x) = x$

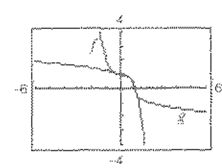
$g(f(x)) = g(9 - x^2) = \sqrt{9 - (9 - x^2)} = \sqrt{x^2} = x$



Reflections in the line $y = x$

19. $f(g(x)) = f\left(\sqrt[3]{1 - x}\right) = 1 - \left(\sqrt[3]{1 - x}\right)^3 = 1 - (1 - x) = x$

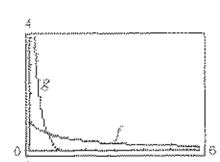
$g(f(x)) = g(1 - x^3) = \sqrt[3]{1 - (1 - x^3)} = \sqrt[3]{x^3} = x$



Reflections in the line $y = x$

20. $f(x) = \dfrac{1}{1 + x},\ x \ge 0;\ g(x) = \dfrac{1 - x}{x},\ 0 < x \le 1$

$$f(g(x)) = f\left(\frac{1 - x}{x}\right) = \frac{1}{1 + \left(\dfrac{1 - x}{x}\right)} = \frac{1}{\dfrac{x}{x} + \dfrac{1 - x}{x}} = \frac{1}{\dfrac{1}{x}} = x$$



Reflections in the line $y = x$

$$g(f(x)) = g\left(\frac{1}{1 + x}\right) = \frac{1 - \left(\dfrac{1}{1 + x}\right)}{\left(\dfrac{1}{1 + x}\right)} = \frac{\dfrac{1 + x}{1 + x} - \dfrac{1}{1 + x}}{\dfrac{1}{1 + x}} = \frac{\dfrac{x}{1 + x}}{\dfrac{1}{1 + x}} = \frac{x}{1 + x} \cdot \frac{x + 1}{1} = x$$

21. The inverse is a line through $(-1, 0)$.

Matches graph (c).

22. The inverse is a line through $(0, 6)$ and $(6, 0)$.

Matches graph (b).

23. The inverse is half a parabola starting at $(1, 0)$.

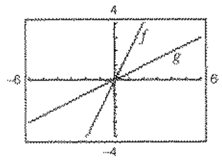
Matches graph (a).

24. The inverse is a reflection in $y = x$ of a third-degree equation through $(0, 0)$.

Matches graph (d).

25. $f(x) = 2x, \quad g(x) = \dfrac{x}{2}$

(a)



Reflection in the line $y = x$

(b)

x	-2	-1	0	1	2
$f(x)$	-4	-2	0	2	4

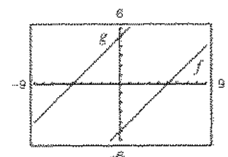
x	-4	-2	0	2	4
$g(x)$	-2	-1	0	1	2

The entries in the tables are the same, except that the rows are interchanged.

26. $f(x) = x - 5$

$g(x) = x + 5$

(a)



The graphs are reflections in the line $y = x$.

(b)

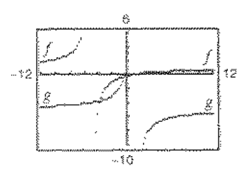
x	-5	-3	0	3	5
$f(x)$	-10	-8	-5	-2	0

x	-10	-8	-5	-2	0
$g(x)$	-5	-3	0	3	5

The entries in the table are the same except that the rows are interchanged.

27. $f(x) = \dfrac{x - 1}{x + 5}, \quad g(x) = -\dfrac{5x + 1}{x - 1} = \dfrac{5x + 1}{1 - x}$

(a)



Reflection in the line $y = x$

(b)

x	-2	-1	0	3	5
$f(x)$	-1	$-\frac{1}{2}$	$-\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{5}$

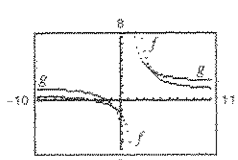
x	-1	$-\frac{1}{2}$	$-\frac{1}{5}$	$\frac{1}{4}$	$\frac{2}{5}$
$g(x)$	-2	-1	0	3	5

The entries in the tables are the same, except that the rows are interchanged.

28. $f(x) = \dfrac{x + 3}{x - 2}$

$g(x) = \dfrac{2x + 3}{x - 1}$

(a)



Reflection in the line $y = x$.

(b)

x	-4	-3	0	3	6
$f(x)$	$\frac{1}{6}$	0	$-\frac{3}{2}$	6	$\frac{9}{4}$

x	$\frac{1}{6}$	0	$-\frac{3}{2}$	6	$\frac{9}{4}$
$g(x)$	-4	-3	0	3	6

The entries in the table are the same except that the rows are interchanged.

29. Not a function

30. It is the graph of a function, but not one-to-one.

31. It is the graph of a one-to-one function.

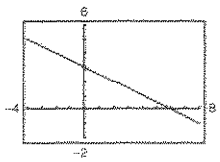
32. It is the graph of a one-to-one function.

33. It is the graph of a one-to-one function.

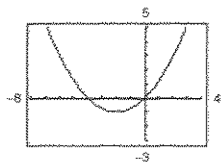
34. It is the graph of a one-to-one function.

35. $f(x) = 3 - \frac{1}{2}x$

f is one-to-one because a horizontal line will intersect the graph at most once.



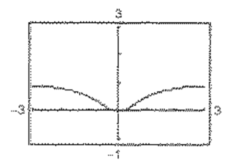
36. $f(x) = \frac{1}{4}(x + 2)^2 - 1$



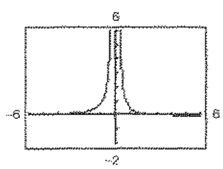
f does not pass the Horizontal Line Test, so f is not one-to-one.

37. $h(x) = \frac{x^2}{x^2 + 1}$

h is not one-to-one because some horizontal lines intersect the graph twice.



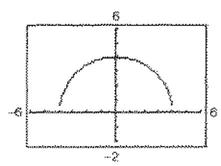
38. $g(x) = \frac{4 - x}{6x^2}$



g does not pass the Horizontal Line Test, so g is not one-to-one.

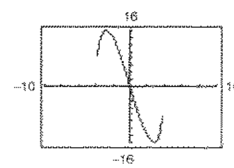
39. $h(x) = \sqrt{16 - x^2}$

h is not one-to-one because some horizontal lines intersect the graph twice.



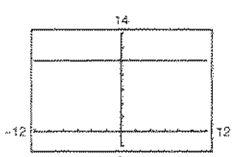
40. $f(x) = -2x\sqrt{16 - x^2}$

is not one-to-one because it does not pass the Horizontal Line Test.



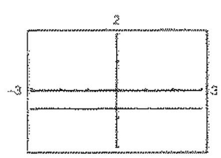
41. $f(x) = 10$

f is not one-to-one because the horizontal line $y = 10$ intersects the graph at every point on the graph.



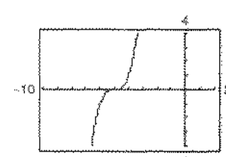
42. $f(x) = -0.65$

is not one-to-one because it does not pass the Horizontal Line Test.



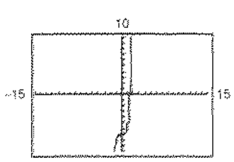
43. $g(x) = (x + 5)^3$

g is one-to-one because a horizontal line will intersect the graph at most once.



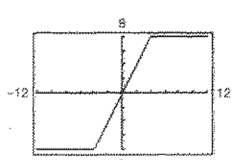
44. $f(x) = x^5 - 7$

is one-to-one because it passes the Horizontal Line Test.



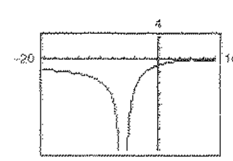
45. $h(x) = |x + 4| - |x - 4|$

h is not one-to-one because some horizontal lines intersect the graph more than once.



46. $f(x) = -\frac{|x - 6|}{|x + 6|}$

is not one-to-one because it does not pass the Horizontal Line Test.



47. $f(x) = x^4$

$$y = x^4$$
$$x = y^4$$
$$y = \pm\sqrt[4]{x}$$

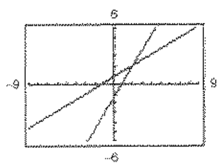
f is not one-to-one.

This does not represent y as a function of x. f does not have an inverse.

48. g is not one-to-one.

For example, $g(1) = g(-1) = 0$.

49.
$$f(x) = \frac{3x + 4}{5}$$
$$y = \frac{3x + 4}{5}$$
$$x = \frac{3y + 4}{5}$$
$$5x = 3y + 4$$
$$5x - 4 = 3y$$
$$\frac{5x - 4}{3} = y$$
$$f^{-1}(x) = \frac{5x - 4}{3}$$



f is one-to-one and has an inverse.

50. $f(x) = 3x + 5$

f is one-to-one.

$$y = 3x + 5$$
$$x = 3y + 5$$
$$x - 5 = 3y$$
$$\frac{x - 5}{3} = y$$
$$f^{-1}(x) = \frac{x - 5}{3}$$

51. $f(x) = \dfrac{1}{x^2}$ is not one-to-one, and does not have an inverse. For example, $f(1) = f(-1) = 1$.

52. $h(x) = \dfrac{4}{x^2}$ is not one-to-one.

For example, $h(1) = h(-1) = 4$.

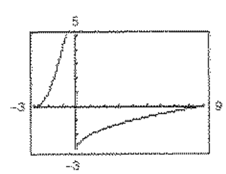
53. $f(x) = (x + 3)^2,\ x \ge -3,\ y \ge 0$

$$y = (x + 3)^2,\ x \ge -3,\ y \ge 0$$
$$x = (y + 3)^2,\ y \ge -3,\ x \ge 0$$
$$\sqrt{x} = y + 3,\ y \ge -3,\ x \ge 0$$
$$y = \sqrt{x} - 3,\ x \ge 0,\ y \ge -3$$

f is one-to-one.

This is a function of x, so f has an inverse.

$f^{-1}(x) = \sqrt{x} - 3,\ x \ge 0$



54. $q(x) = (x - 5)^2,\ x \le 5$ is one-to-one.

$$y = (x - 5)^2,\ x \le 5$$
$$x = (y - 5)^2,\ y \le 5$$
$$-\sqrt{x} = y - 5,\ y \le 5$$
$$y = -\sqrt{x} + 5$$

The inverse is $q^{-1}(x) = -\sqrt{x} + 5$.

55. $f(x) = \sqrt{2x+3} \implies x \ge -\frac{3}{2},\ y \ge 0$

$$y = \sqrt{2x+3},\ x \ge -\frac{3}{2},\ y \ge 0$$

$$x = \sqrt{2y+3},\ y \ge -\frac{3}{2},\ x \ge 0$$

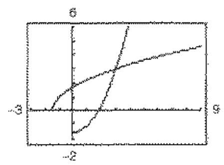
$$x^2 = 2y+3,\ x \ge 0,\ y \ge -\frac{3}{2}$$

$$y = \frac{x^2-3}{2},\ x \ge 0,\ y \ge -\frac{3}{2}$$

f is one to one.

This is a function of x, so f has an inverse.

$$f^{-1}(x) = \frac{x^2-3}{2},\ x \ge 0$$



56. $f(x) = \sqrt{x-2} \implies x \ge 2,\ y \ge 0$

$$y = \sqrt{x-2},\ x \ge 2,\ y \ge 0$$

$$x = \sqrt{y-2},\ y \ge 2,\ x \ge 0$$

$$x^2 = y-2,\ x \ge 0,\ y \ge 2$$

$$x^2 + 2 = y,\ x \ge 0,\ y \ge 2$$

f is one-to-one, so f has an inverse.

$$f^{-1}(x) = x^2 + 2,\ x \ge 0$$

57. $f(x) = |x-2|,\ x \le 2, y \ge 0$

$$y = |x-2|$$

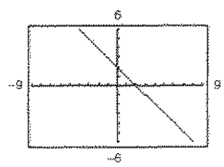
$$x = |y-2|,\quad y \le 2,\quad x \ge 0$$

$$x = -(y-2) \text{ since } y-2 \le 0.$$

$$x = -y+2$$

$$y = -x+2,\quad x \ge 0,\quad y \le 2$$

$$f^{-1}(x) = -x+2,\quad x \ge 0$$



58. $f(x) = \dfrac{x^2}{x^2+1}$

f is not one-to-one.
For instance $f(1) = f(-1)$.
Hence, f does not have an inverse.

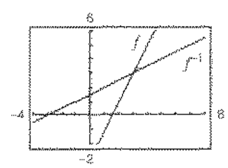
59. $f(x) = 2x-3$

$$y = 2x-3$$

$$x = 2y-3$$

$$y = \frac{x+3}{2}$$

$$f^{-1}(x) = \frac{x+3}{2}$$



Reflections in the line $y = x$

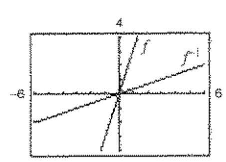
60. $f(x) = 3x$

$$y = 3x$$

$$x = 3y$$

$$\frac{x}{3} = y$$

$$f^{-1}(x) = \frac{x}{3}$$



Reflections in the line $y = x$

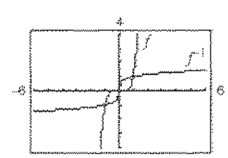
61. $f(x) = x^5$

$$y = x^5$$

$$x = y^5$$

$$y = \sqrt[5]{x}$$

$$f^{-1}(x) = \sqrt[5]{x}$$



Reflections in the line $y = x$

62. $f(x) = x^3 + 1$

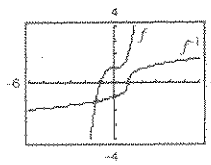
$y = x^3 + 1$

$x = y^3 + 1$

$x - 1 = y^3$

$\sqrt[3]{x - 1} = y$

$f^{-1}(x) = \sqrt[3]{x - 1}$



Reflections in the line $y = x$

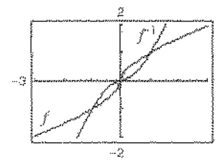
63. $f(x) = x^{3/5}$

$y = x^{3/5}$

$x = y^{3/5}$

$y = x^{5/3}$

$f^{-1}(x) = x^{5/3}$



Reflections in the line $y = x$

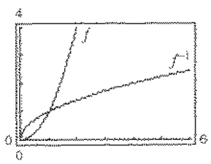
64. $f(x) = x^2, x \geq 0$

$y = x^2$

$x = y^2$

$\sqrt{x} = y$

$f^{-1}(x) = \sqrt{x}$



Reflections in the line $y = x$

65. $f(x) = \sqrt{4 - x^2}, 0 \leq x \leq 2$

$y = \sqrt{4 - x^2}$

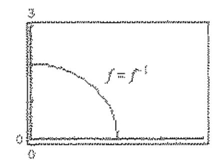
$x = \sqrt{4 - y^2}$

$x^2 = 4 - y^2$

$y^2 = 4 - x^2$

$y = \sqrt{4 - x^2}$

$f^{-1}(x) = \sqrt{4 - x^2}, 0 \leq x \leq 2$



Reflections in the line $y = x$

66. $f(x) = \sqrt{16 - x^2}, \quad -4 \leq x \leq 0$

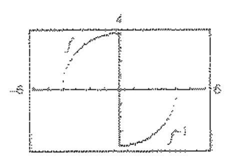
$y = \sqrt{16 - x^2}$

$x = \sqrt{16 - y^2}, -4 \leq y \leq 0$

$x^2 = 16 - y^2$

$y^2 = 16 - x^2$

$y = -\sqrt{16 - x^2}, \ 0 \leq x \leq 4$



67. $f(x) = \dfrac{4}{x}$

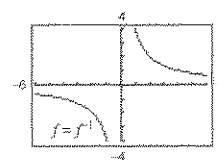
$y = \dfrac{4}{x}$

$x = \dfrac{4}{y}$

$xy = 4$

$y = \dfrac{4}{x}$

$f^{-1}(x) = \dfrac{4}{x}$



Reflections in the line $y = x$

68. $f(x) = \dfrac{6}{\sqrt{x}}$

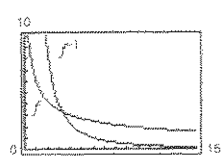
$y = \dfrac{6}{\sqrt{x}}$

$x = \dfrac{6}{\sqrt{y}}$

$x^2 = \dfrac{36}{y}$

$y = \dfrac{36}{x^2}, \ x > 0$

$f^{-1}(x) = \dfrac{36}{x^2}, \ x > 0$



69. If we let $f(x) = (x - 2)^2$, $x \geq 2$, then f has an inverse. [**Note:** We could also let $x \leq 2$.]

$f(x) = (x - 2)^2, \ x \geq 2, \ y \geq 0$

$y = (x - 2)^2, \ x \geq 2, \ y \geq 0$

$x = (y - 2)^2, \ x \geq 0, \ y \geq 2$

$\sqrt{x} = y - 2, \ x \geq 0, \ y \geq 2$

$\sqrt{x} + 2 = y, \ x \geq 0, \ y \geq 2$

Thus, $f^{-1}(x) = \sqrt{x} + 2, \ x \geq 0.$

70. If we let $f(x) = 1 - x^4$, $x \ge 0$, then f has an inverse. [**Note:** We could also let $x \le 0$.]

$$f(x) = 1 - x^4,\ x \ge 0 \Rightarrow y \le 1$$

$$y = 1 - x^4,\ x \ge 0,\ y \le 1$$

$$x = 1 - y^4,\ y \ge 0,\ x \le 1$$

$$y^4 = 1 - x,\ y \ge 0,\ x \le 1$$

$$y = \sqrt[4]{1 - x},\ x \le 1,\ y \ge 0$$

Thus, $f^{-1}(x) = \sqrt[4]{1 - x}$, $x \le 1$.

71. If we let $f(x) = |x + 2|$, $x \ge -2$, then f has an inverse. [**Note:** We could also let $x \le -2$.]

$$f(x) = |x + 2|,\quad x \ge -2$$

$$f(x) = x + 2 \text{ when } x \ge -2$$

$$y = x + 2,\ x \ge -2,\quad y \ge 0$$

$$x = y + 2,\ x \ge 0,\quad y \ge -2$$

$$x - 2 = y,\quad x \ge 0,\ y \ge -2$$

Thus, $f^{-1}(x) = x - 2$, $x \ge 0$.

72. If we let $f(x) = |x - 2|$, $x \ge 2$, then f has an inverse. [**Note:** We could also let $x \le 2$.]

$$f(x) = |x - 2|,\ x \ge 2$$

$$f(x) = x - 2 \text{ when } x \ge 2.$$

$$y = x - 2,\ x \ge 2,\ y \ge 0$$

$$x = y - 2,\ x \ge 0,\ y \ge 2$$

$$x + 2 = y,\ x \ge 0,\ y \ge 2$$

Thus, $f^{-1}(x) = x + 2$, $x \ge 0$.

73. Let $f(x) = (x + 3)^2$, $x \ge -3$.

$$y = (x + 3)^2$$

$$x = (y + 3)^2$$

$$\sqrt{x} = y + 3$$

$$y = \sqrt{x} - 3$$

$$f^{-1}(x) = \sqrt{x} - 3$$

Domain f: $x \ge -3$ Range f: $y \ge 0$

Domain f^{-1}: $x \ge 0$ Range f^{-1}: $y \ge -3$

74. Let $f(x) = (x - 4)^2$, $x \ge 4$.

$$y = (x - 4)^2$$

$$x = (y - 4)^2$$

$$\sqrt{x} = y - 4$$

$$y = \sqrt{x} + 4$$

$$f^{-1}(x) = \sqrt{x} + 4$$

Domain f: $x \ge 4$ Range f: $y \ge 0$

Domain f^{-1}: $x \ge 0$ Range f^{-1}: $y \ge 4$

75. Let $f(x) = -2x^2 + 5$, $x \ge 0$.

$$y = -2x^2 + 5$$

$$x = -2y^2 + 5$$

$$x - 5 = -2y^2$$

$$y^2 = \frac{x - 5}{-2} = \frac{5 - x}{2}$$

$$y = \sqrt{(5 - x)/2}$$

$$f^{-1}(x) = \sqrt{\frac{5 - x}{2}}$$

Domain f: $x \ge 0$ Range f: $y \le 5$

Domain f^{-1}: $x \le 5$ Range f^{-1}: $y \ge 0$

76. Let $f(x) = \frac{1}{2}x^2 - 1$, $x \ge 0$.

$$y = \tfrac{1}{2}x^2 - 1$$

$$x = \tfrac{1}{2}y^2 - 1$$

$$2(x + 1) = y^2$$

$$f^{-1}(x) = \sqrt{2x + 2}$$

Domain f: $x \ge 0$ Range f: $y \ge -1$

Domain f^{-1}: $x \ge -1$ Range f^{-1}: $y \ge 0$

77. Let $f(x) = |x - 4| + 1$, $x \ge 4$ and $y \ge 1$.

$$y = |x - 4| + 1$$

$$y = x - 3 \text{ because } x \ge 4.$$

$$x = y - 3$$

$$y = x + 3$$

$$f^{-1}(x) = x + 3,\quad x \ge 1$$

Domain f: $x \ge 4$ Range f: $y \ge 1$

Domain f^{-1}: $x \ge 1$ Range f^{-1}: $y \ge 4$

78. Let $f(x) = -|x-1| - 2, \quad x \geq 1$ and $y \leq -2$.

$$y = -|x-1| - 2 = -(x-1) - 2 \text{ because } x \geq 1.$$
$$y = -x - 1$$
$$x = -y - 1$$
$$x + 1 = -y$$
$$f^{-1}(x) = -x - 1, \quad x \leq -2$$

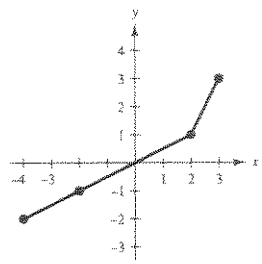
Domain f: $x \geq 1$ Range f: $y \leq -2$

Domain f^{-1}: $x \leq -2$ Range f^{-1}: $y \geq 1$

79.

x	$f(x)$
-2	-4
-1	-2
1	2
3	3

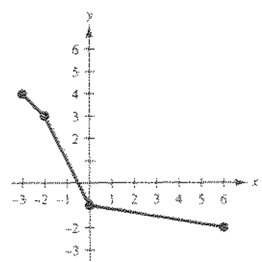
x	$f^{-1}(x)$
-4	-2
-2	-1
2	1
3	3



80.

x	$f(x)$
4	-3
3	-2
-1	0
-2	6

x	$f^{-1}(x)$
-3	4
-2	3
0	-1
6	-2



81. $f^{-1}(0) = \frac{1}{2}$ because $f\left(\frac{1}{2}\right) = 0$.

82. $g^{-1}(0) = -2$ because $g(-2) = 0$.

83. $(f \circ g)(2) = f(3) = -2$

84. $g(f(-4)) = g(4) = 6$

85. $f^{-1}(g(0)) = f^{-1}(2) = 0$

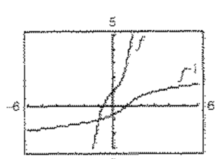
86. $(g^{-1} \circ f)(3) = g^{-1}(-2) = -3$

87. $(g \circ f^{-1})(2) = g(0) = 2$

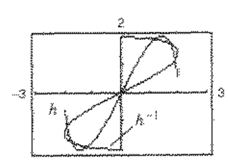
88. $(f^{-1} \circ g^{-1})(-2) = f^{-1}(-3) = 1$

89. $f(x) = x^3 + x + 1$

The graph of the inverse relation is an inverse function since it satisfies the Vertical Line Test.



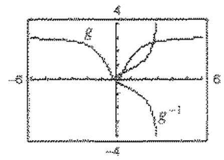
90. (a) and (b)



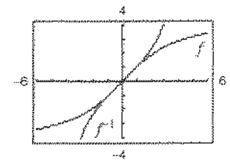
(c) Not an inverse function since it does not satisfy the Vertical Line Test.

91. $g(x) = \dfrac{3x^2}{x^2+1}$

The graph of the inverse relation is not an inverse function since it does not satisfy the Vertical Line Test.



92. (a) and (b)



(c) Inverse function since it satisfies the Vertical Line Test.

In Exercises 93–98, $f(x) = \frac{1}{8}x - 3$, $f^{-1}(x) = 8(x+3)$, $g(x) = x^3$, $g^{-1}(x) = \sqrt[3]{x}$.

93. $(f^{-1} \circ g^{-1})(1) = f^{-1}(g^{-1}(1)) = f^{-1}(\sqrt[3]{1}) = 8(\sqrt[3]{1} + 3) = 8(1+3) = 32$

94. $(g^{-1} \circ f^{-1})(-3) = g^{-1}(f^{-1}(-3)) = g^{-1}(8(-3+3)) = g^{-1}(0) = \sqrt[3]{0} = 0$

95. $(f^{-1} \circ f^{-1})(6) = f^{-1}(f^{-1}(6)) = f^{-1}(8[6+3]) = f^{-1}(72) = 8(72+3) = 600$

96. $(g^{-1} \circ g^{-1})(-4) = g^{-1}(g^{-1}(-4)) = g^{-1}(\sqrt[3]{-4}) = \sqrt[3]{\sqrt[3]{-4}} = -\sqrt[9]{4}$

97. $(fg)(x) = f(g(x)) = f(x^3) = \frac{1}{8}x^3 - 3$

Now find the inverse of $(f \circ g)(x) = \frac{1}{8}x^3 - 3$:

$$\begin{aligned} y &= \tfrac{1}{8}x^3 - 3 \\ x &= \tfrac{1}{8}y^3 - 3 \\ x + 3 &= \tfrac{1}{8}y^3 \\ 8(x+3) &= y^3 \\ \sqrt[3]{8(x+3)} &= y \\ (f \circ g)^{-1}(x) &= 2\sqrt[3]{x+3} \end{aligned}$$

Note: $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$

98.
$$\begin{aligned} (g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(8(x+3)) \\ &= \sqrt[3]{8(x+3)} \\ &= 2\sqrt[3]{x+3} \end{aligned}$$

In Exercises 99–102, $f(x) = x + 4$, $f^{-1}(x) = x - 4$, $g(x) = 2x - 5$, $g^{-1}(x) = \dfrac{x+5}{2}$.

99.
$$\begin{aligned} (g^{-1} \circ f^{-1})(x) &= g^{-1}(f^{-1}(x)) \\ &= g^{-1}(x-4) \\ &= \frac{(x-4)+5}{2} \\ &= \frac{x+1}{2} \end{aligned}$$

100.
$$\begin{aligned} (f^{-1} \circ g^{-1})(x) &= f^{-1}(g^{-1}(x)) \\ &= f^{-1}\left(\frac{x+5}{2}\right) \\ &= \frac{x+5}{2} - 4 \\ &= \frac{x+5-8}{2} \\ &= \frac{x-3}{2} \end{aligned}$$

101. $(f \circ g)(x) = f(g(x)) = f(2x - 5) = (2x - 5) + 4 = 2x - 1$. Now find the inverse of $(f \circ g)(x) = 2x - 1$:

$$y = 2x - 1$$
$$x = 2y - 1$$
$$x + 1 = 2y$$
$$y = \frac{x+1}{2}$$
$$(f \circ g)^{-1}(x) = \frac{x+1}{2}$$

Note that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$; see Exercise 99.

102. $(g \circ f)(x) = g(f(x)) = g(x + 4) = 2(x + 4) - 5 = 2x + 8 - 5 = 2x + 3$. Now find inverse:

$$y = 2x + 3$$
$$x = 2y + 3$$
$$x - 3 = 2y$$
$$\frac{x-3}{2} = y$$
$$(g \circ f)^{-1}(x) = \frac{x-3}{2}$$

Note that $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$.

103. (a) Yes, f is one-to-one. For each European shoe size, there is exactly one U.S. shoe size.

(b) $f(11) = 45$

(c) $f^{-1}(43) = 10$ because $f(10) = 43$.

(d) $f(f^{-1}(41)) = f(8) = 41$

(e) $f^{-1}(f(13)) = f^{-1}(47) = 13$

104. (a) Yes, g is one-to-one. For each European shoe size, there is exactly one U.S. shoe size.

(b) $g(6) = 38$

(c) $g^{-1}(42) = 9$ because $g(9) = 42$.

(d) $g(g^{-1}(39)) = g(7) = 39$

(e) $g^{-1}(g(5)) = g^{-1}(37) = 5$

105. (a) Yes, f is one-to-one, so f^{-1} exists.

(b) f^{-1} gives the year corresponding to the 10 values in the second column.

(c) $f^{-1}(650.3) = 10$ because $f(10) = 650.3$.

(d) No, because $f(11) = f(15) = 690.4$.

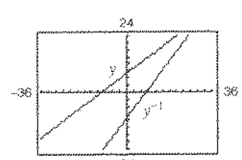
106. (a)

$$y = 8 + 0.75x$$
$$x = 8 + 0.75y$$
$$x - 8 = 0.75y$$
$$\frac{x-8}{0.75} = y$$
$$y = f^{-1}(x) = \frac{x-8}{0.75}$$

$x =$ hourly wage

$y =$ number of units produced

(b)



(c) If 10 units are produced, then

$y = 8 + 0.75(10) = \$15.50$.

(d) If the hourly wage is \$22.25, then

$$y = \frac{22.25 - 8}{0.75} = 19 \text{ units.}$$

107. False. $f(x) = x^2$ is even, but f^{-1} does not exist.

108. True. If $(0, b)$ is the y-intercept of f, then $(b, 0)$ is the x-intercept of f^{-1}.

109. We will show that $(f \circ g)^{-1}(x) = (g^{-1} \circ f^{-1})(x)$ for all x in their domains.
Let $y = (f \circ g)^{-1}(x) \implies (f \circ g)(y) = x$ then $f(g(y)) = x \implies f^{-1}(x) = g(y)$.
Hence, $(g^{-1} \circ f^{-1})(x) = g^{-1}(f^{-1}(x)) = g^{-1}(g(y)) = y = (f \circ g)^{-1}(x)$.
Thus, $g^{-1} \circ f^{-1} = (f \circ g)^{-1}$.

110. If f is one-to-one, then f^{-1} exists. If f is odd, then $f(-x) = -f(x)$. Consider $f(x) = y \leftrightarrow f^{-1}(y) = x$.
Then $f^{-1}(-y) = f^{-1}(-f(x)) = f^{-1}(f(-x)) = -x = -f^{-1}(y)$. Thus, f^{-1} is odd.

111. No, the graphs are not reflections of each other in the line $y = x$.

112. Yes, the graphs are reflections of each other in the line $y = x$.

113. Yes, the graphs are reflections of each other in the line $y = x$.

114. Yes, the graphs are reflections of each other in the line $y = x$.

115. Yes. The inverse would give the time it took to complete n miles.

116. Yes, assuming that the population is increasing between 1960 and 2005. The inverse would give the year corresponding to a given population.

117. No. The function oscillates.

118. No, because heights remain constant, or even decrease, after many years.

119. $\dfrac{27x^3}{3x^2} = 9x,\ x \neq 0$

120. $\dfrac{5x^2y}{xy + 5x} = \dfrac{5x^2y}{x(y + 5)} = \dfrac{5xy}{y + 5},\ x \neq 0$

121. $\dfrac{x^2 - 36}{6 - x} = \dfrac{(x - 6)(x + 6)}{-(x - 6)} = \dfrac{x + 6}{-1} = -x - 6,\ x \neq 6$

122. $\dfrac{x^2 + 3x - 40}{x^2 - 3x - 10} = \dfrac{(x - 5)(x + 8)}{(x - 5)(x + 2)} = \dfrac{x + 8}{x + 2},\ x \neq 5$

123. $4x - y = 3$

$y = 4x - 3$

Yes, y is a function of x.

124. $x = 5$. No. Does not pass Vertical Line Test

125. $x^2 + y^2 = 9$

$y = \pm\sqrt{9 - x^2}$

No, y is not a function of x.

126. $x^2 + y = 8$

$y = -x^2 + 8$

Yes, y is a function of x.

127. $y = \sqrt{x + 2}$

Yes, y is a function of x.

128. $x - y^2 = 0$

$y^2 = x$

$y = \pm\sqrt{x}$

No, y is not a function of x.

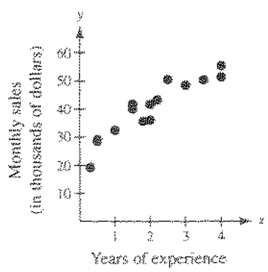
Section 1.7 Linear Models and Scatter Plots

- You should know how to construct a scatter plot for a set of data
- You should recognize if a set of data has a positive correlation, negative correlation, or neither.
- You should be able to fit a line to data using the point-slope formula.
- You should be able to use the regression feature of a graphing utility to find a linear model for a set of data.
- You should be able to find and interpret the correlation coefficient of a linear model.

Vocabulary Check

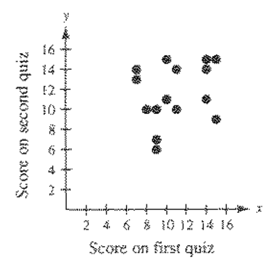
1. positive
2. negative
3. fitting a line to data
4. $-1, 1$

1. (a)



(b) Yes, the data appears somewhat linear. The more experience, x, corresponds to higher sales, y.

2. (a)



(b) No. Quiz scores are dependent on several variables, such as study time, class attendance, etc.

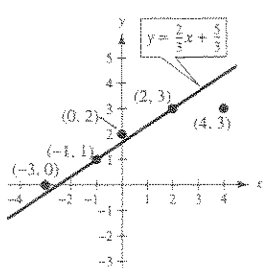
3. Negative correlation—y decreases as x increases.

4. No correlation

5. No correlation

6. Positive correlation

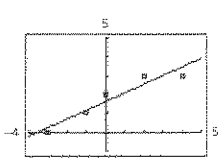
7. (a)



(b) $y = 0.46x + 1.62$

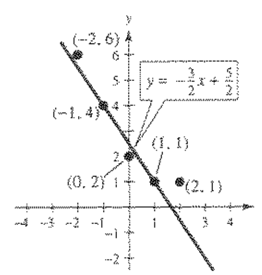
Correlation coefficient: 0.95095

(c)



(d) Yes, the model appears valid.

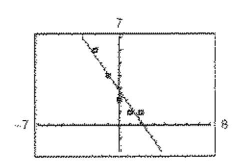
8. (a)



(b) $y = -1.3x + 2.8$

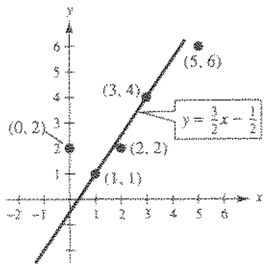
Correlation coefficient: -0.94812

(c)



(d) The model appears valid.

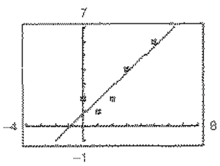
9. (a)



(b) $y = 0.95x + 0.92$

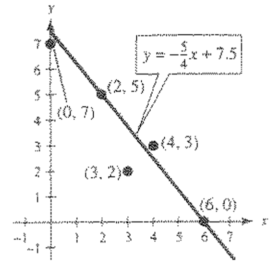
Correlation coefficient: 0.90978

(c)



(d) Yes, the model appears valid.

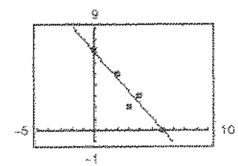
10. (a)



(b) $y = -1.15x + 6.85$

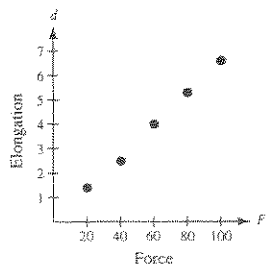
Correlation coefficient: -0.95175

(c)



(d) The model is somewhat valid.

11. (a)

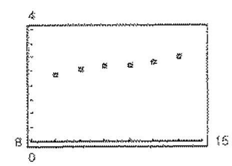


(b) $d = 0.07F - 0.3$

(c) $d = 0.066F$ or $F = 15.13d + 0.096$

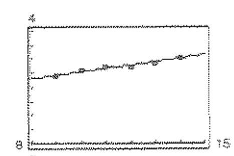
(d) If $F = 55$, $d = 0.066(55) \approx 3.63$ cm.

12. (a)



(b) $y = 0.122t + 1.32$

(c)



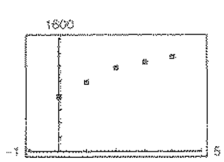
Yes, the model is a good fit.

(d) For 2010, $t = 20$ and $y \approx 3.76$ minutes.

For 2015, $t = 25$ and $y \approx 4.37$ minutes.

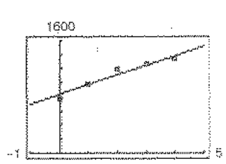
Yes, the answers seem reasonable.

13. (a)



(b) $y = 136.1t + 836$

(c)



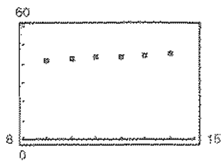
Yes, the model is a good fit.

(d) For 2005, $t = 5$ and $y \approx 1516.5$, or \$1,516,500.

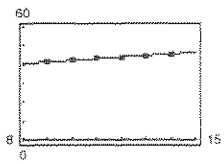
For 2010, $t = 10$ and $y \approx 2197$, or \$2,197,000.

Yes, the answers seem reasonable.

(e) The slope is 136.1. It says that the mean salary increases by \$136,100 per year.

14. (a) 

(b) $y = 0.84t + 33.9$

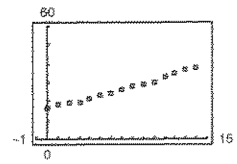
(c) 

Yes, the model is a good fit.

(d) For 2005, $t = 15$ and $y \approx 46.5$, or \$46,500.

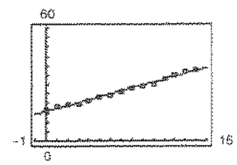
For 2010, $t = 20$ and $y \approx 50.7$, or \$50,700.

Yes, the answers seem reasonable.

15. (a) 

(b) $C = 1.552t + 15.70$

Correlation coefficient: 0.99544

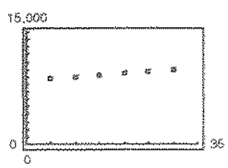
(c) 

(d) The model is a good fit.

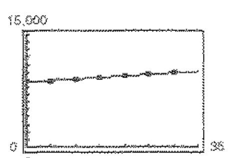
(e) For 2005, $t = 15$, $y_1 \approx \$38.98$.

For 2010, $t = 20$, $y_1 \approx \$46.74$.

(f) Answers will vary.

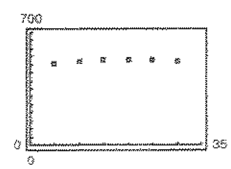
16. (a) 

(b) $P = 42.0t + 8585$

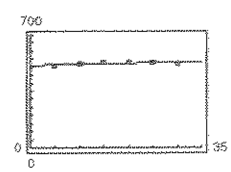
(c) 

The model is a good fit.

(d) For 2050, $t = 50$ and $P = 10{,}685$, or 10,685,000 people. Answers will vary.

17. (a) 

(b) $P = 0.6t + 512$

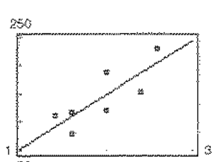
(c) 

The model is not a good fit.

(d) For 2050, $t = 50$ and $P = 542$, or 542,000 people. Answers will vary.

18. (a) $y = 47.77x + 103.8$

Correlation coefficient: 0.81238

(b) 

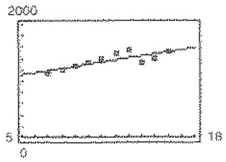
(c) The slope represents the increase in sales due to increased advertising.

(d) For \$1500, $x = 1.5$ and $y = 175.455$ or \$175,455.

19. (a) $T = 36.7t + 926$

Correlation coefficient: 0.79495

(b)



(c) The slope indicated the number of new stores opened per year.

(d) $T = 36.7t + 926 > 1800$

$36.7t > 874$

$t > 23.8$

The number of stores will exceed 1800 near the end of 2013.

(e)

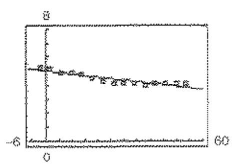
Year	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
Data	1130	1182	1243	1307	1381	1475	1553	1308	1400	1505
Model	1183	1220	1256	1293	1330	1366	1403	1440	1477	1513

The model is not a good fit, especially around $t = 14$.

20. (a) $y = -0.022t + 5.03$

(b) The negative slope indicates that the times are decreasing.

(c)



(d) The model is not very accurate.

(e) Answers will vary.

21. True. To have positive correlation, the y-values tend to increase as x increases.

22. False. The closer to 1 or -1, the better the fit.

23. Answers will vary.

24. Answers will vary.

25. $f(x) = 2x^2 - 3x + 5$

(a) $f(-1) = 2 + 3 + 5 = 10$

(b) $f(w + 2) = 2(w + 2)^2 - 3(w + 2) + 5$

$= 2w^2 + 5w + 7$

26. $g(x) = 5x^2 - 6x + 1$

(a) $g(-2) = 5(4) - 6(-2) + 1 = 33$

(b) $g(z - 2) = 5(z - 2)^2 - 6(z - 2) + 1$

$= 5z^2 - 26z + 33$

27. $h(x) = \begin{cases} 1 - x^2, & x \le 0 \\ 2x + 3, & x > 0 \end{cases}$

(a) $h(1) = 2(1) + 3 = 5$ (b) $h(0) = 1 - 0 = 1$

28. (a) $k(-3) = 5 - 2(-3) = 11$

(b) $k(-1) = (-1)^2 + 4 = 5$

29. $6x + 1 = -9x - 8$

$15x = -9$

$x = -\frac{9}{15} = -\frac{3}{5}$

30. $3(x - 3) = 7x + 2$

$-11 = 4x$

$x = -\frac{11}{4}$

31. $8x^2 - 10x - 3 = 0$

$(4x + 1)(2x - 3) = 0$

$x = -\frac{1}{4}, \frac{3}{2}$

32. $10x^2 - 23x - 5 = 0$

$(2x - 5)(5x + 1) = 0$

$x = \frac{5}{2}, -\frac{1}{5}$

33. $2x^2 - 7x + 4 = 0$

$x = \dfrac{7 \pm \sqrt{49 - 4(4)(2)}}{4}$

$= \dfrac{7 \pm \sqrt{17}}{4}$

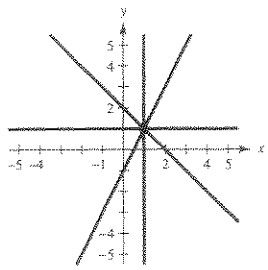
34. $2x^2 - 8x + 5 = 0$

$x = \dfrac{8 \pm \sqrt{64 - 40}}{4}$

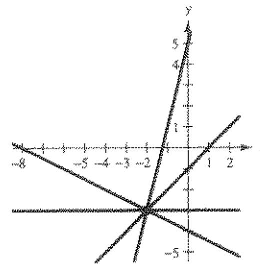
$= 2 \pm \dfrac{\sqrt{6}}{2}$

Review Exercises for Chapter 1

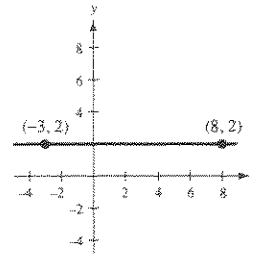
1.



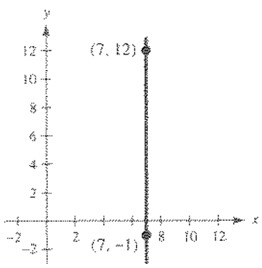
2.



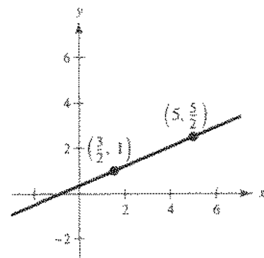
3. $m = \dfrac{2-2}{8-(-3)} = \dfrac{0}{11} = 0$



4. Slope $= \dfrac{12-(-1)}{7-7}$, undefined

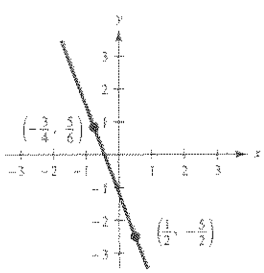


5. $m = \dfrac{(5/2)-1}{5-(3/2)} = \dfrac{3/2}{7/2} = \dfrac{3}{7}$



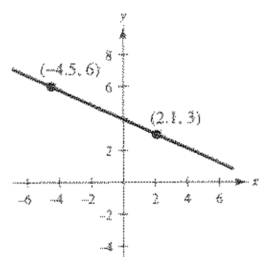
6. Slope $= \dfrac{\frac{5}{6}-\left(-\frac{5}{2}\right)}{-\frac{3}{4}-\frac{1}{2}} = \dfrac{\frac{5}{6}+\frac{15}{6}}{-\frac{3}{4}-\frac{2}{4}} = \dfrac{\frac{10}{3}}{-\frac{5}{4}}$

$= -\dfrac{10}{3}\cdot\dfrac{4}{5} = -\dfrac{8}{3}$

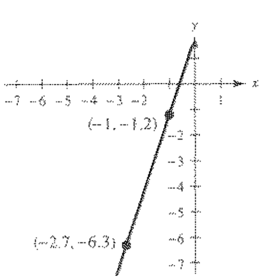


7. $(-4.5, 6), (2.1, 3)$

$m = \dfrac{3-6}{2.1-(-4.5)} = \dfrac{-3}{6.6} = -\dfrac{30}{66} = -\dfrac{5}{11}$



8. Slope $= \dfrac{-1.2+6.3}{-1+2.7} = \dfrac{5.1}{1.7} = \dfrac{51}{17} = 3$



9. (a) $y + 1 = \dfrac{1}{4}(x-2)$

$4y + 4 = x - 2$

$-x + 4y + 6 = 0$

(b) Three additional points:

$(2+4, -1+1) = (6, 0)$

$(6+4, 0+1) = (10, 1)$

$(10+4, 1+1) = (14, 2)$

(other answers possible)

10. (a)
$$y - 5 = -\tfrac{3}{2}(x + 3)$$
$$2y - 10 = -3x - 9$$
$$3x + 2y - 1 = 0$$

(b) Three additional points:

$(-3 + 2, 5 - 3) = (-1, 2)$

$(-1 + 2, 2 - 3) = (1, -1)$

$(1 + 2, -1 - 3) = (3, -4)$

(other answers possible)

11. (a)
$$y + 5 = \tfrac{3}{2}(x - 0)$$
$$2y + 10 = 3x$$
$$-3x + 2y + 10 = 0$$

(b) Three additional points:

$(0 + 2, -5 + 3) = (2, -2)$

$(2 + 2, -2 + 3) = (4, 1)$

$(4 + 2, 1 + 3) = (6, 4)$

(other answers possible)

12. (a)
$$y - 0 = -\tfrac{2}{3}(x - 3)$$
$$3y = -2x + 6$$
$$2x + 3y - 6 = 0$$

(b) Three additional points:

$(3 - 3, 0 + 2) = (0, 2)$

$(0 - 3, 2 + 2) = (-3, 4)$

$(-3 - 3, 4 + 2) = (-6, 6)$

(other answers possible)

13. (a)
$$y + 5 = -1\left(x - \tfrac{1}{5}\right)$$
$$y + 5 = -x + \tfrac{1}{5}$$
$$5y + 25 = -5x + 1$$
$$5x + 5y + 24 = 0$$

(b) Three additional points:

$\left(\frac{1}{5} + 1, -5 - 1\right) = \left(\frac{6}{5}, -6\right)$

$\left(\frac{6}{5} + 1, -6 - 1\right) = \left(\frac{11}{5}, -7\right)$

$\left(\frac{11}{5} + 1, -7 - 1\right) = \left(\frac{16}{5}, -8\right)$

(other answers possible)

14. (a)
$$y - \tfrac{7}{8} = -\tfrac{4}{5}(x - 0)$$
$$40y - 35 = -32x$$
$$32x + 40y - 35 = 0$$

(b) Three additional points:

$\left(0 + 5, \frac{7}{8} - 4\right) = \left(5, -\frac{25}{8}\right)$

$\left(5 + 5, -\frac{25}{8} - 4\right) = \left(10, -\frac{57}{8}\right)$

$\left(10 + 5, -\frac{57}{8} - 4\right) = \left(15, -\frac{89}{8}\right)$

(other answers possible)

15. (a) $y - 6 = 0(x + 2)$

$y - 6 = 0$

(b) Three additional points:

$(0, 6), (1, 6), (2, 6)$

(other answers possible)

16. (a) $y - 8 = 0(x + 8) = 0$

$y = 8$ (horizontal line)

$y - 8 = 0$

(b) Three additional points: $(0, 8), (1, 8), (2, 8)$

(other answers possible)

17. (a) m is undefined means that the line is vertical.

$x - 10 = 0$

(b) Three additional points: $(10, 0), (10, 1), (10, 2)$

(other answers possible)

18. (a) Slope is undefined, line is vertical: $x = 5$ or $x - 5 = 0$

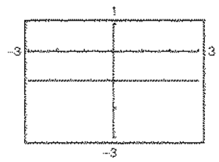
(b) Three additional points: $(5, 0), (5, 1), (5, 2)$

(other answers possible)

19. $y + 1 = \dfrac{-1 + 1}{4 - 2}(x - 2)$

$= 0(x - 2) = 0 \implies y = -1$

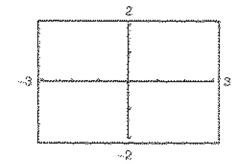
(Slope $= 0$)



20. Slope is undefined.

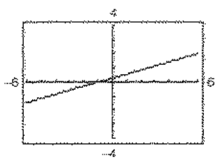
Line is vertical.

$x = 0$



21. $y - 0 = \dfrac{2 - 0}{6 - (-1)}(x + 1)$

$= \dfrac{2}{7}(x + 1) = \dfrac{2}{7}x + \dfrac{2}{7} \implies y = \dfrac{2}{7}x + \dfrac{2}{7}$



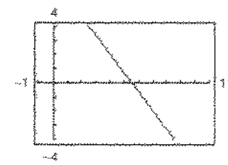
22. $y - 6 = \dfrac{6 - 2}{1 - 4}(x - 1)$

$y - 6 = \dfrac{4}{-3}(x - 1)$

$-3y + 18 = 4x - 4$

$-3y = 4x - 22$

$y = -\dfrac{4}{3}x + \dfrac{22}{3}$



23. $t = 8$ corresponds to 2008.

Point: (8, 12,500), slope: 850

$V - 12{,}500 = 850(t - 8)$

$V = 850t + 5700$

24. $m = -115$

Point: (8, 3795)

$V - 3795 = -115(t - 8)$

$V = -115t + 4715$

25. $m = 42.70$

Point: (8, 625.50)

$V - 625.50 = 42.70(t - 8)$

$V = 42.70t + 283.90$

26. $t = 8$ corresponds to 2008.

Point: (8, 72.95), slope: -5.15

$V - 72.95 = -5.15(t - 8)$

$V = -5.15t + 114.15$

27. (2, 160,000), (3, 185,000)

$m = \dfrac{185{,}000 - 160{,}000}{3 - 2} = 25{,}000$

$S - 160{,}000 = 25{,}000(t - 2)$

$S = 25{,}000t + 110{,}000$

For the fourth quarter let $t = 4$. Then we have

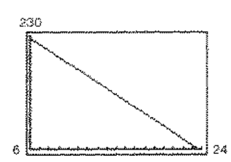
$S = 25{,}000(4) + 110{,}000 = \$210{,}000$.

28. (a) Point: (6, 225), slope: -12.75

$V - 225 = -12.75(t - 6)$

$V = -12.75t + 301.5$

(b)



(c) In 2010, $t = 10$ and $V = 174$ dollars.

(d) $V = 0$ when $t \approx 23.6$, (2023).

Algebraically,

$V = -12.75t + 301.5 = 0$

$t = \dfrac{301.5}{12.75} \approx 23.6.$

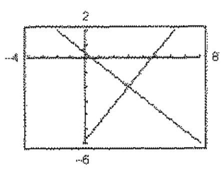
29. $5x - 4y = 8 \implies y = \frac{5}{4}x - 2$ and $m = \frac{5}{4}$

(a) Parallel slope: $m = \frac{5}{4}$

$$y - (-2) = \frac{5}{4}(x - 3)$$
$$4y + 8 = 5x - 15$$
$$0 = 5x - 4y - 23$$
$$y = \frac{5}{4}x - \frac{23}{4}$$

(b) Perpendicular slope: $m = -\frac{4}{5}$

$$y - (-2) = -\frac{4}{5}(x - 3)$$
$$5y + 10 = -4x + 12$$
$$4x + 5y - 2 = 0$$
$$y = -\frac{4}{5}x + \frac{2}{5}$$



30. Slope of given line: $m = -\frac{2}{3}$

(a) $y - 3 = -\frac{2}{3}(x + 8) \implies 3y - 9 = -2x - 16$

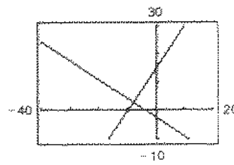
$\implies 2x + 3y + 7 = 0$

$$y = -\frac{2}{3}x - \frac{7}{3}$$

(b) $y - 3 = \frac{3}{2}(x + 8) \implies 2y - 6 = 3x + 24$

$\implies 3x - 2y + 30 = 0$

$$y = \frac{3}{2}x + 15$$



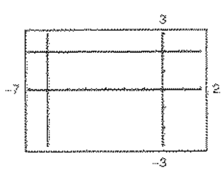
31. $x = 4$ is a vertical line; the slope is not defined.

(a) Parallel line: $x = -6$

(b) Perpendicular slope: $m = 0$

Perpendicular line: $y - 2 = 0(x + 6)$

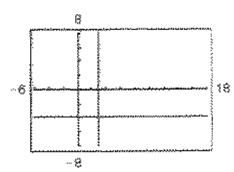
$= 0 \implies y = 2$



32. $y = 2$ is a horizontal line.

(a) Parallel line through $(3, -4)$: $y = -4$

(b) Perpendicular line through $(3, -4)$: $x = 3$



33. (a) Not a function. 20 is assigned two different values.

(b) Function

(c) Function

(d) Not a function. No value is assigned to 30.

34. (a) Not a function. u is assigned two different values.

(b) Function

(c) Function

(d) Not a function. w is assigned two different values and u is unassigned.

35. No, y is not a function of x. Some x-values correspond to two y-values. For example, $x = 1$ corresponds to $y = 4$ and $y = -4$.

36. Yes, $y = 2x - 3$.

37. $y = \sqrt{1 - x}$

Each x value, $x \le 1$, corresponds to only one y-value so y is a function of x.

38. No, does not pass Vertical Line Test.

39. $f(x) = x^2 + 1$

(a) $f(1) = 1^2 + 1 = 2$

(b) $f(-3) = (-3)^2 + 1 = 10$

(c) $f(b^3) = (b^3)^2 + 1 = b^6 + 1$

(d) $f(x - 1) = (x - 1)^2 + 1 = x^2 - 2x + 2$

40. $g(x) = x^{4/3}$

(a) $g(8) = 8^{4/3} = 2^4 = 16$

(b) $g(t + 1) = (t + 1)^{4/3}$

(c) $g(-27) = (-27)^{4/3} = (-3)^4 = 81$

(d) $g(-x) = (-x)^{4/3} = x^{4/3}$

41. $h(x) = \begin{cases} 2x + 1, & x \le -1 \\ x^2 + 2, & x > -1 \end{cases}$

(a) $h(-2) = 2(-2) + 1 = -3$

(b) $h(-1) = 2(-1) + 1 = -1$

(c) $h(0) = 0^2 + 2 = 2$

(d) $h(2) = 2^2 + 2 = 6$

42. $f(x) = \dfrac{3}{2x - 5}$

(a) $f(1) = \dfrac{3}{2(1) - 5} = -1$

(b) $f(-2) = \dfrac{3}{2(-2) - 5} = \dfrac{3}{-9} = -\dfrac{1}{3}$

(c) $f(t) = \dfrac{3}{2t - 5}$

(d) $f(10) = \dfrac{3}{2(10) - 5} = \dfrac{3}{15} = \dfrac{1}{5}$

43. The domain of $f(x) = \dfrac{x - 1}{x + 2}$ is all real numbers $x \neq -2$.

44. The domain of $f(x) = \dfrac{x^2}{x^2 + 1}$ is the set of all real numbers.

45. $f(x) = \sqrt{25 - x^2}$

Domain: $25 - x^2 \ge 0$

$(5 + x)(5 - x) \ge 0$

Domain: $[-5, 5]$

46. The domain of $f(x) = \sqrt{x^2 - 16}$ is given by

$x^2 - 16 \ge 0$

$x^2 \ge 16.$

The domain is $(-\infty, -4] \cup [4, \infty)$.

47. The domain of $g(5) = \dfrac{5s + 5}{3s - 9}$ is all real numbers $s \neq 3$.

48. The domain of $f(x) = \dfrac{2x + 1}{3x + 4}$ is all real numbers $\neq -\dfrac{4}{3}$.

49. (a) $C(x) = 16{,}000 + 5.35x$

(b) $P(x) = R(x) - C(x)$

$= 8.20x - (16{,}000 + 5.35x)$

$= 2.85x - 16{,}000$

50. $R(t)$ in billions of dollars

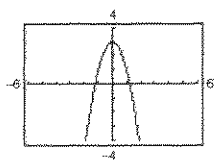
Year	1997	1998	1999	2000	2001	2002	2003	2004
$R(t)$	6.744	7.744	8.996	10.5	12.699	11.994	10.448	8.929

51.
$$
\begin{aligned}
f(x) &= 2x^2 + 3x - 1\\
f(x+h) &= 2(x+h)^2 + 3(x+h) - 1\\
&= 2x^2 + 4xh + 2h^2 + 3x + 3h - 1\\
\frac{f(x+h)-f(x)}{h} &= \frac{(2x^2 + 4xh + 2h^2 + 3x + 3h - 1) - (2x^2 + 3x - 1)}{h}\\
&= \frac{4xh + 2h^2 + 3h}{h}\\
&= 4x + 2h + 3, \quad h \neq 0
\end{aligned}
$$

52.
$$
\begin{aligned}
f(x+h) &= (x+h)^3 - 5(x+h)^2 + (x+h)\\
&= x^3 + 3x^2h + 3xh^2 + h^3 - 5x^2 - 10xh - 5h^2 + x + h\\
f(x+h) - f(x) &= 3x^2h + 3xh^2 + h^3 - 10xh - 5h^2 + h\\
\frac{f(x+h)-f(x)}{h} &= \frac{h(3x^2 + 3xh + h^2 - 10x - 5h + 1)}{h}\\
&= 3x^2 + 3xh + h^2 - 10x - 5h + 1, \quad h \neq 0
\end{aligned}
$$

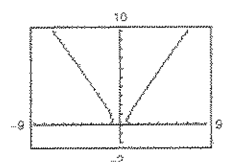
53. Domain: All real numbers

Range: $y \le 3$



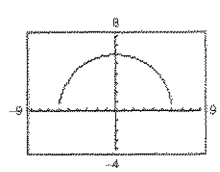
54. Domain: $2x^2 - 1 \ge 0 \implies x^2 \ge \frac{1}{2} \implies \left(-\infty, -\frac{\sqrt{2}}{2}\right] \cup \left[\frac{\sqrt{2}}{2}, \infty\right)$

Range: $[0, \infty)$



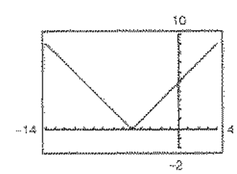
55. Domain: $36 - x^2 \ge 0 \implies x^2 \le 36 \implies -6 \le x \le 6$

Range: $0 \le y \le 6$



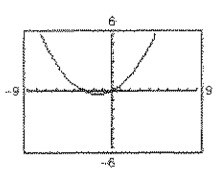
56. Domain: all real numbers

Range: $[0, \infty)$



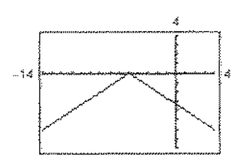
57. (a) $y = \dfrac{x^2 + 3x}{6}$

(b) y is a function of x.



58. (a) $y = -\frac{2}{3}|x + 5|$

(b) y is a function of x.

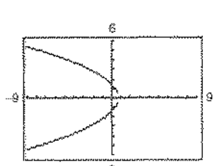


59. (a) $3x + y^2 = 2$

$y^2 = 2 - 3x$

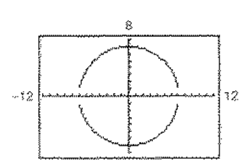
$y = \pm\sqrt{2 - 3x}$

(b) y is not a function of x.

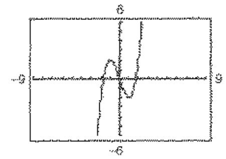


60. (a) $x^2 + y^2 = 49$

(b) y is not a function of x.



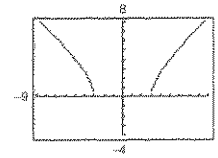
61. $f(x) = x^3 - 3x$

(a) 

(b) Increasing on $(-\infty, -1)$ and $(1, \infty)$

Decreasing on $(-1, 1)$

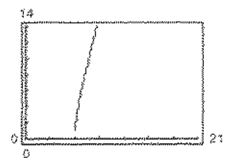
62. $f(x) = \sqrt{x^2 - 9}$

(a) 

(b) Increasing on $(3, \infty)$

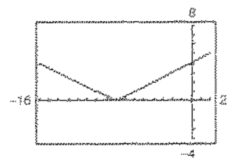
Decreasing on $(-\infty, -3)$

63. $f(x) = x\sqrt{x - 6}$

(a) 

(b) Increasing on $(6, \infty)$

64. $f(x) = \dfrac{|x + 8|}{2}$

(a) 

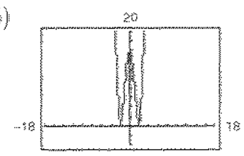
(b) Increasing on $(-8, \infty)$

Decreasing on $(-\infty, -8)$

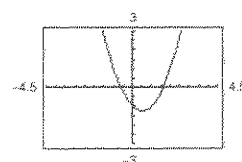
65. $f(x) = (x^2 - 4)^2$

Relative minima: $(-2, 0)$ and $(2, 0)$

Relative maximum: $(0, 16)$

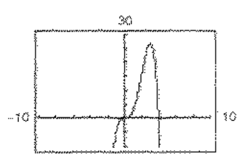


66. $f(x) = x^2 - x - 1$



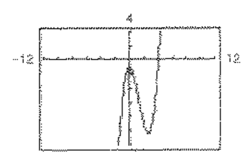
Relative minimum: $(0.5, -1.25)$

67. $h(x) = 4x^3 - x^4$



Relative maximum: $(3, 27)$

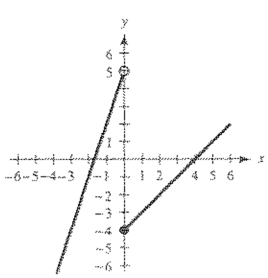
68. $f(x) = x^3 - 4x^2 - 1$



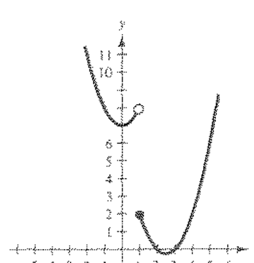
Relative maximum: $(0, -1)$

Relative minimum: $(2.67, -10.48)$

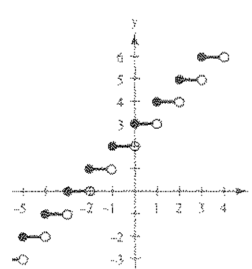
69. $f(x) = \begin{cases} 3x + 5, & x < 0 \\ x - 4, & x \geq 0 \end{cases}$



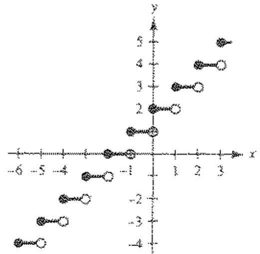
70. $f(x) = \begin{cases} x^2 + 7, & x < 1 \\ x^2 - 5x + 6, & x \geq 1 \end{cases}$



71. $f(x) = [\![x]\!] + 3$



72. $f(x) = [\![x + 2]\!]$



73. $f(-x) = (-x)^2 + 6$

$= x^2 + 6$

$= f(x)$

Even

74. $f(-x) = (-x)^2 - (-x) - 1$

$= x^2 + x - 1$

$\neq f(x)$

and $f(-x) \neq -f(x)$

Neither even nor odd

75. $f(-x) = ((-x)^2 - 8)^2$

$= (x^2 - 8)^2$

$= f(x)$

f is even.

76. $f(x) = 2x^3 - x^2$ is neither even nor odd.

77. $f(-x) = 3(-x)^{5/2} \neq f(x)$ and $f(-x) \neq -f(x)$

Neither even nor odd

(Note that the domain of f is $x \geq 0$.)

78. $f(-x) = 3(-x)^{2/5} = 3x^{2/5} = f(x)$

Even

79. $f(x) = -2$ is a constant function.

80. $g(x) = x$ is the parent function. f is obtained from g by a reflection in the x-axis, followed by a vertical shift five units upward.

$f(x) = -x + 5 = -g(x) + 5$

81. $g(x) = x^2$ is the parent function. f is obtained from g by a horizontal shift two units to the right, followed by a vertical shift one unit upward.

$f(x) = (x - 2)^2 + 1 = g(x - 2) + 1$

82. $g(x) = -x^3 - 2$ is obtained from $f(x) = x^3$ by a reflection in the x-axis, followed by a vertical shift two units downward.

$g(x) = -f(x) - 2$

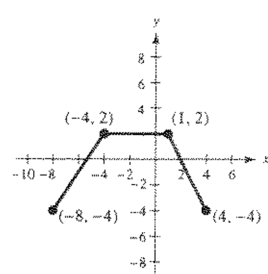
83. $g(x) = |x| + 3$ is obtained from $f(x) = |x|$ by a vertical shift three units upward.

$g(x) = f(x) + 3$

84. $g(x) = -\sqrt{x - 3}$ is obtained from $f(x) = \sqrt{x}$ by a horizontal shift three units to the right followed by a reflection in the x-axis.

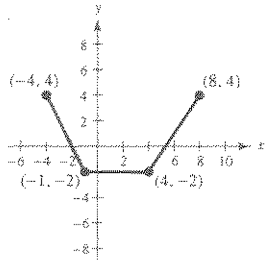
$g(x) = -f(x - 3)$

85.



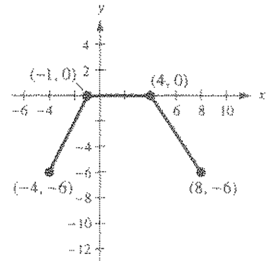
$y = f(-x)$ is a reflection in the y-axis.

86.



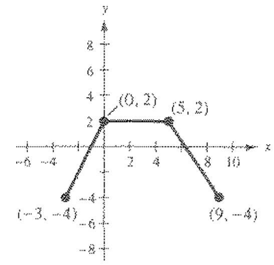
$y = -f(x)$ is a reflection in the x-axis.

87.



$y = f(x) - 2$ is a vertical shift two units downward.

88.

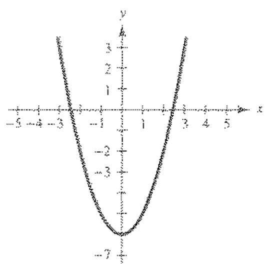


$y = f(x - 1)$ is a horizontal shift one unit to the right.

89. (a) $f(x) = x^2$

(b) h is a vertical shift six units downward.

(c)

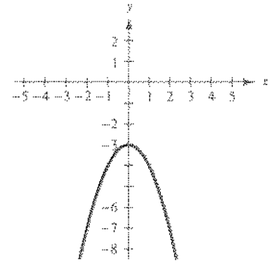


(d) $h(x) = f(x) - 6$

90. (a) $f(x) = x^2$

(b) h is a reflection in the x-axis, followed by a vertical shift three units downward.

(c)



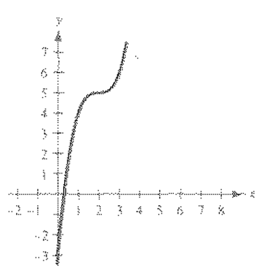
(d) $h(x) = -f(x) - 3$

91. $h(x) = (x - 2)^3 + 5$

(a) $f(x) = x^3$

(b) The graph of h is a horizontal shift of f two units to the right, followed by a vertical shift five units upward.

(c)



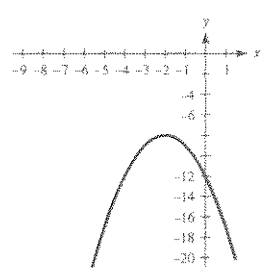
(d) $h(x) = (x - 2)^3 + 5 = f(x - 2) + 5$

92. $h(x) = -(x + 2)^2 - 8$

(a) $f(x) = x^2$

(b) The graph of h is a horizontal shift of f two units to the left, followed by a reflection in the x-axis, followed by a vertical shift eight units downward.

(c)

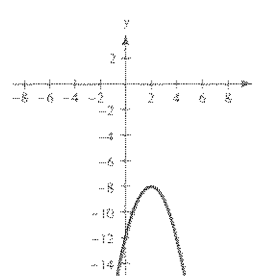


(d) $h(x) = -(x + 2)^2 - 8 = -f(x + 2) - 8$

93. (a) $f(x) = x^2$

(b) h is a horizontal shift two units to the right, a reflection in the x-axis, followed by a vertical shift eight units downward.

(c)



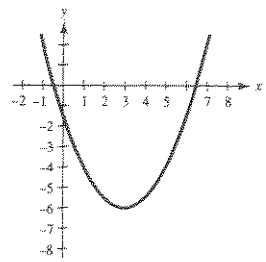
(d) $h(x) = -f(x - 2) - 8$

94. $h(x) = \frac{1}{2}(x - 3)^2 - 6$

(a) $f(x) = x^2$

(b) The graph of h is a horizontal shift of f three units to the right, followed by a vertical shrink of $\frac{1}{2}$, followed by a vertical shift six units downward.

(c)



(d) $h(x) = \frac{1}{2}(x - 3)^2 - 6$

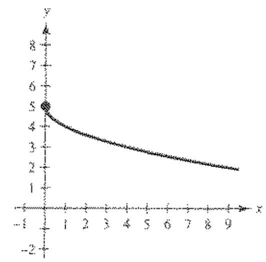
$= \frac{1}{2}f(x - 3) - 6$

95. $h(x) = -\sqrt{x} + 5$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a reflection of f in the x-axis, followed by a vertical shift five units upward.

(c)



(d) $h(x) = -\sqrt{x} + 5$

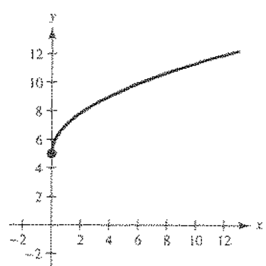
$= -f(x) + 5$

96. $h(x) = 2\sqrt{x} + 5$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a vertical stretch of f of 2, followed by a vertical shift five units upward.

(c)



(d) $h(x) = 2\sqrt{x} + 5$

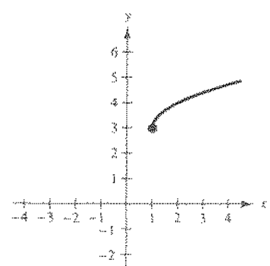
$= 2f(x) + 5$

97. $h(x) = \sqrt{x - 1} + 3$

(a) $f(x) = \sqrt{x}$

(b) The graph of h is a horizontal shift of one unit to the right, followed by a vertical shift three units upward.

(c)



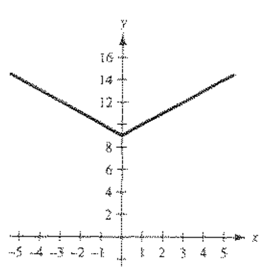
(d) $h(x) = f(x - 1) + 3$

98. $h(x) = |x| + 9$

(a) $f(x) = |x|$

(b) The graph of h is a vertical shift of f nine units upward.

(c)



(d) $h(x) = |x| + 9$

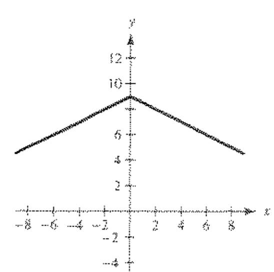
$= f(x) + 9$

99. $h(x) = -\frac{1}{2}|x| + 9$

(a) $f(x) = |x|$

(b) h is a vertical shrink, followed by a reflection in the x-axis, followed by a vertical shift nine units upward.

(c)



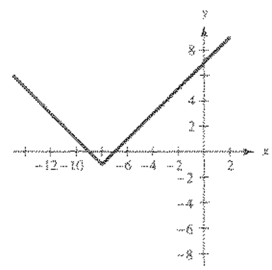
(d) $h(x) = -\frac{1}{2}f(x) + 9$

100. $h(x) = |x + 8| - 1$

(a) $f(x) = |x|$

(b) h is a horizontal shift eight units to the left, followed by a vertical shift one unit downward.

(c)



(d) $h(x) = f(x + 8) - 1$

101. $(f - g)(4) = f(4) - g(4)$
$= [3 - 2(4)] - \sqrt{4}$
$= -5 - 2$
$= -7$

102. $(f + h)(5) = f(5) + h(5)$
$= -7 + 77$
$= 70$

103. $(f + g)(25) = f(25) + g(25)$
$= -47 + 5$
$= -42$

104. $(g - h)(1) = g(1) - h(1) = 1 - 5 = -4$

105. $(fh)(1) = f(1)h(1) = (3 - 2(1))(3(1)^2 + 2)$
$= (1)(5) = 5$

106. $\left(\frac{g}{h}\right)(1) = \frac{g(1)}{h(1)} = \frac{1}{5}$

107. $(h \circ g)(7) = h(g(7))$
$= h(\sqrt{7})$
$= 3(\sqrt{7})^2 + 2$
$= 23$

108. $(g \circ f)(-2) = g(7) = \sqrt{7}$

109. $(f \circ h)(-4) = f(h(-4))$
$= f(50)$
$= -97$

110. $(g \circ h)(6) = g(h(6))$
$= g(110)$
$= \sqrt{110}$

111. $f(x) = x^2, g(x) = x + 3$
$(f \circ g)(x) = f(x + 3)$
$= (x + 3)^2 = h(x)$

112. $f(x) = x^3, g(x) = 1 - 2x$
$(f \circ g)(x) = f(1 - 2x) = (1 - 2x)^3 = h(x)$

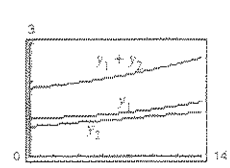
113. $f(x) = \sqrt{x}, g(x) = 4x + 2$
$(f \circ g)(x) = f(4x + 2) = \sqrt{4x + 2} = h(x)$

114. $f(x) = \sqrt[3]{x}, g(x) = (x + 2)^2$
$(f \circ g)(x) = f((x + 2)^2) = \sqrt[3]{(x + 2)^2} = h(x)$

115. $f(x) = \frac{4}{x}, g(x) = x + 2$
$(f \circ g)(x) = f(x + 2) = \frac{4}{x + 2} = h(x)$

116. $f(x) = \frac{6}{x^3}, g(x) = 3x + 1$
$(f \circ g)(x) = f(3x + 1) = \frac{6}{(3x + 1)^3} = h(x)$

117.



118. $y_1 + y_2 = (0.00204t^2 + 0.0015t + 1.021) + (0.0274t + 0.785)$

For 2008, let $t = 18$.

$(y_1 + y_2)(18) \approx 2.987$, or about 2,987,000 students.

119. $f(x) = 6x$

$f^{-1}(x) = \frac{1}{6}x$

$f(f^{-1}(x)) = f\left(\frac{1}{6}x\right) = 6\left(\frac{1}{6}x\right) = x$

$f^{-1}(f(x)) = f^{-1}(6x) = \frac{1}{6}(6x) = x$

120. $f(x) = x + 5$

$f^{-1}(x) = x - 5$

$f(f^{-1}(x)) = f(x - 5) = (x - 5) + 5 = x$

$f^{-1}(f(x)) = f^{-1}(x + 5) = (x + 5) - 5 = x$

121. $f(x) = \frac{1}{2}x + 3 \implies f^{-1}(x) = 2(x - 3) = 2x - 6$

$$f(f^{-1}(x)) = f(2(x - 3)) = \frac{1}{2}(2(x - 3)) + 3 = x - 3 + 3 = x$$

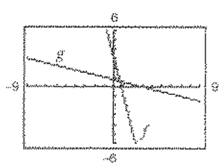
$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{2}x + 3\right) = 2\left(\frac{1}{2}x + 3 - 3\right) = 2\left(\frac{1}{2}x\right) = x$$

122. $f(x) = \frac{x - 4}{5} \implies f^{-1}(x) = 5x + 4$

$$f(f^{-1}(x)) = f(5x + 4) = \frac{5x + 4 - 4}{5} = \frac{5x}{5} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{x - 4}{5}\right) = 5\left(\frac{x - 4}{5}\right) + 4 = x - 4 + 4 = x$$

123. (a)



Reflection in the line $y = x$

(b)

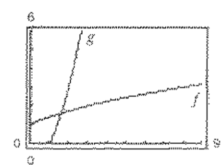
x	-5	-1	0	1	3
$f(x)$	23	7	3	-1	-9

x	23	7	3	-1	-9
$g(x)$	-5	-1	0	1	3

The entries in the table are the same except that their rows are interchanged.

124. $f(x) = \sqrt{x + 1}, \quad g(x) = x^2 - 1, \quad x \geq 0$

(a)



Reflections in $y = x$

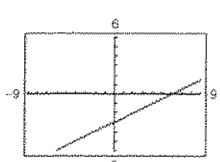
(b)

x	-1	0	3	8	15
$f(x)$	0	1	2	3	4

x	0	1	2	3	4
$g(x)$	-1	0	3	8	15

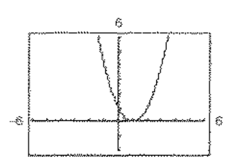
The entries are the same, except that the rows are interchanged.

125.



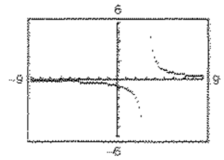
$f(x) = \frac{1}{2}x - 3$ passes the Horizontal Line Test, and hence is one-to-one and has an inverse $(f^{-1}(x) = 2(x + 3))$.

126.



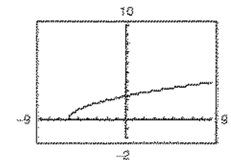
$f(x) = (x - 1)^2$ does not pass the Horizontal Line Test. Not one-to-one

127.



$h(t) = \dfrac{2}{t-3}$ passes the Horizontal Line Test, and hence is one-to-one.

128.



$g(x) = \sqrt{x+6}$ passes the Horizontal Line Test. It is one-to-one.

129.
$$y = \frac{1}{2}x - 5$$
$$x = \frac{1}{2}y - 5$$
$$x + 5 = \frac{1}{2}y$$
$$y = 2(x+5)$$
$$f^{-1}(x) = 2x + 10$$

130.
$$f(x) = \frac{7x+3}{8}$$
$$y = \frac{1}{8}(7x+3)$$
$$x = \frac{1}{8}(7y+3)$$
$$8x = 7y + 3$$
$$8x - 3 = 7y$$
$$f^{-1}(x) = \frac{1}{7}(8x - 3)$$

131.
$$f(x) = 4x^3 - 3$$
$$y = 4x^3 - 3$$
$$x = 4y^3 - 3$$
$$x + 3 = 4y^3$$
$$\frac{x+3}{4} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x+3}{4}}$$

132.
$$y = 5x^3 + 2$$
$$x = 5y^3 + 2$$
$$x - 2 = 5y^3$$
$$\frac{x-2}{5} = y^3$$
$$f^{-1}(x) = \sqrt[3]{\frac{x-2}{5}}$$

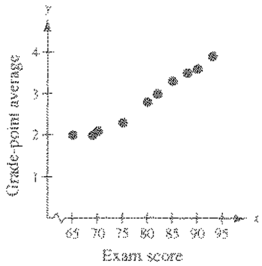
133.
$$f(x) = \sqrt{x+10}$$
$$y = \sqrt{x+10}, x \geq -10, y \geq 0$$
$$x = \sqrt{y+10}, y \geq -10, x \geq 0$$
$$x^2 = y + 10$$
$$x^2 - 10 = y$$
$$f^{-1}(x) = x^2 - 10, x \geq 0$$

134.
$$f(x) = 4\sqrt{6-x},\ x \leq 6, y \geq 0$$
$$y = 4\sqrt{6-x}$$
$$x = 4\sqrt{6-y},\ y \leq 6, x \geq 0$$
$$x^2 = 16(6-y) = 96 - 16y$$
$$16y = 96 - x^2$$
$$y = \frac{96 - x^2}{16}$$
$$f^{-1}(x) = \frac{96 - x^2}{16},\ x \geq 0$$

135. Negative correlation

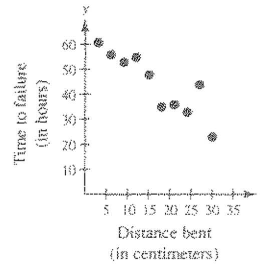
136. No correlation

137. (a)



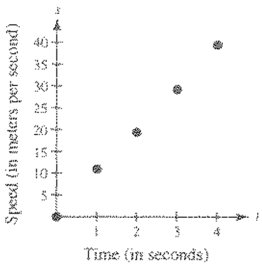
(b) Yes, the relationship is approximately linear. Higher entrance exam scores, x, are associated with higher grade-point averages, y.

138. (a)



(b) Answers will vary.

139. (a)



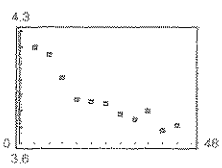
(b) $s \approx 10t$ (Approximations will vary.)

(c) $s = 9.7t + 0.4$; 0.99933

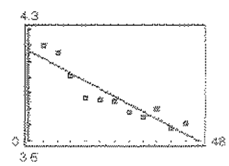
(d) For $t = 2.5$, $S \approx 24.7$ m/sec.

140. (a) $y = -0.0119t + 4.164$, linear model; -0.91997

(b)



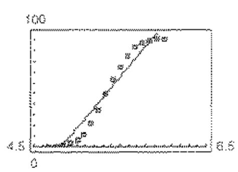
(c)



(d), (e) Answers will vary.

141. $y = 95.174x - 458.423$

142. $y = 95.174x - 458.423$



143. The model does not fit well.

144. No. The data stops at (6.00, 100.0).

145. False. $g(x) = -[(x - 6)^2 + 3] = -(x - 6)^2 - 3$ and $g(-1) = -52 \neq 28$

146. True. $f^{-1}(x) = x^{1/n}$, n odd

147. False. $f(x) = \dfrac{1}{x}$ or $f(x) = x$ satisfies $f = f^{-1}$.

148. False. The slope can be positive, negative, or 0.

Chapter 1 Practice Test

1. Find the slope of the line passing through the points $(-2, 2)$ and $(1, 3)$.

2. Find an equation for the line passing through the points $(3, -2)$ and $(4, -5)$. Use a graphing utility to sketch a graph of the line.

3. Find an equation of the line that passes through the point $(-1, 5)$ and has slope -3. Use a graphing utility to sketch a graph of the line.

4. Find the slope-intercept form of the line that passes through the point $(-3, 2)$ and is perpendicular to $3x + 5y = 7$.

5. Does the equation $x^4 + y^4 = 16$ represent y as a function of x?

6. Evaluate the function $f(x) = |x - 2|/(x - 2)$ at the points $x = 0$, $x = 2$, and $x = 4$.

7. Find the domain of the function $f(x) = 5/(x^2 - 16)$.

8. Find the domain of the function $g(t) = \sqrt{4 - t}$.

9. Use a graphing utility to sketch the graph of the function $f(x) = 3 - x^6$ and determine if the function is even, odd, or neither.

10. Determine the open interval(s) on which the function $f(x) = 12x - x^3$ is increasing.

11. Use a graphing utility to approximate any relative minimum or maximum values of the function $y = 4 - x + x^3$.

12. Compare the graph of $f(x) = x^3 - 3$ with the graph of $y = x^3$.

13. Compare the graph of $f(x) = \sqrt{x - 6}$ with the graph of $y = \sqrt{x}$.

14. Find $g \circ f$ if $f(x) = \sqrt{x}$ and $g(x) = x^2 - 2$. What is the domain of $g \circ f$?

15. Find f/g if $f(x) = 3x^2$ and $g(x) = 16 - x^4$. What is the domain of f/g?

16. Show that $f(x) = 3x + 1$ and $g(x) = \dfrac{x - 1}{3}$ are inverse functions algebraically and graphically.

17. Find the inverse of $f(x) = \sqrt{9 - x^2}$, $0 \le x \le 3$. Graph f and f^{-1} in the same viewing rectangle.

18. Use a graphing utility to find the least squares regression line for the points $(-1, 0)$, $(0, 1)$, $(3, 3)$, $(4, 5)$. Graph the points and the line.